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Decision Aiding

Ordinal judgments in multiattribute decision analysis

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Abstract

The article discusses the contradiction between the ambiguity of human judgment in a multicriterion environment and the exactness of the assessments required in the majority of the decision-making methods. Preferential information from the decision makers in the ordinal form (e.g., "more preferable", "less preferable", etc.) is argued to be more stable and more reliable than cardinal input. Ways of obtaining and using ordinal judgments for rank ordering of multiattribute alternatives are discussed. The effectiveness of the step-wise procedure of using ordinal tradeoffs for comparison of alternatives is evaluated. We introduce the notion of ordinal tradeoffs, presentation of ordinal tradeoffs as a flexible three-stage process, a paired joint ordinal scale (PJOS), and evaluation of the effectiveness of the three-stage process. Simulation results examine the sensitivity of the number of pairwise comparisons required for given numbers of criteria and categories within criteria, as well as the number of alternatives analyzed. This simulation shows that ordinal pairwise comparisons provide sufficient power to discriminate between 75% and 80% of the alternatives compared. While the proportional number of pairwise comparisons relative to the maximum possible decreases with the number of criteria and categories, the method is relatively insensitive to the number of alternatives considered. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Many decision problems involve the need to trade off multiple criteria. Decision aids are technologies designed to help people learn more about decision choices and their tradeoffs. This paper discusses the use of a joint ordinal scale (JOS) to reflect decision maker preference without having to develop a scaled utility or value function.

One of the most popular approaches in this field is multiattribute utility theory (MAUT) which is often substituted for by multiattribute value theory for practical tasks under certainty (Keeney and Raiffa, 1976; Von Winterfeldt and Edwards, 1986). This approach is rather straightforward. However, numeric values for scores of each alternative on each attribute *and* relative

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weights reflecting attribute scales need to be established. While relative weights are established by decision maker indication of indifference, this only works by presenting the decision maker with numbers. Past work (Slovic et al., 1977; Kahneman et al., 1982; Keeney and Raiffa, 1976 and others) demonstrated that the process of eliciting the necessary information for such a decision was one of the major challenges facing the field. Experiments (such as Larichev, 1992; Larichev et al., 1995) have shown the limited capacities of people in providing quantitative information for some tasks. Ordinal input is less complex, and thus we expect it to more accurately reflect decision maker preference.

It is not always necessary to replace qualitative information with quantitative measures. If we do not require the complete rank order of alternatives (ordering the n alternatives 1 through n), but rather want to narrow a subset of the better alternatives in a partial rank order (where each rank may have a number of alternatives assigned), the need for exact numerical values may disappear. For example, the majority of approaches in mathematical programming with multiple criteria provide the decision maker with the structure of the efficient set of alternatives. Then, in a dialog with the decision maker about possible directions for alternatives' adjustment, try to form the best solution (Wierzbicki, 1982; Korhonen et al., 1984; Steuer, 1986; Korhonen, 1988; Olson, 1992; Lofti et al., 1992).

Outranking methods (Roy, 1968; Brans and Vincke, 1985; Salminen et al., 1989) combine each alternative's advantages and disadvantages in two respective criteria (concordance and discordance indices), and present the decision maker with subgroups of incomparable better alternatives for further analysis. Such multiattribute systems as ARIADNE and HIPRE 3+ (Olson, 1996) try to allow the decision maker to introduce possible intervals for criterion weights, providing maximum and minimum possible values for each alternative. As with the outranking approaches, these methods can result in a subset of overlapping (incomparable) alternatives.

In all of these methods, the decision maker is provided with the partial results, evaluates them, and decides on further actions (change the aspiration levels, adjust the discordance/concordance indices, change the evaluation criteria, etc.).

Once we decide that the complete order of alternatives is not necessary, we are able to discuss methods and approaches to elicit required decision makers' preference information in a verbal (usually ordinal) form. There have been a number of attempts to aid decisions using the decision maker's ordinal preferences (see, e.g., Fishburn, 1964; Barron, 1973; Kirkwood and Sarin, 1985; Salminen et al., 1989; Cook and Kress, 1992; Barron, 1992; Edwards and Barron, 1994; Larichev and Moshkovich, 1997). What is new in this paper is not comparison of quantitative and qualitative preferences, but rather a structured overview of approaches within the qualitative field, an introduction of the notion of ordinal tradeoffs, presentation of ordinal tradeoffs as a flexible three-stage process, a paired joint ordinal scale (PJOS), and evaluation of the effectiveness of the three-stage process.

In Section 2 the main approaches to elicitation of attribute weights and values for possible attribute levels will be analyzed. Directions in ordinal information implementation will be discussed and procedures for ordinal tradeoffs for comparison of multiattribute alternatives will be proposed. Simulation results based on the elicitation of ordinal preference information will be described in Section 4, followed by conclusions.

2. Problems of preference elicitation in the construction of multiattribute value functions

Problems of elicitation and implementation of information about the decision maker's preferences in comparison and evaluation of alternatives have become very important in the process of developing methods for multicriteria decision making. The most popular multiattribute value model in practical cases is the additive model (see, e.g. Von Winterfeldt and Edwards, 1986; Watson and Buede, 1987; Corner and Kirkwood, 1991; Keeney, 1992). Within the framework of the additive model the task under consideration may be presented as follows:

Given:

1. $K = \{q_i\},\ i = 1, 2, \ldots, Q$ – a set of attributes upon which alternatives are evaluated.
2. n_q – the number of possible levels on the scale of the qth attribute ($q \in K$).
3. $X_q = \{x_{iq}\}$ – a set of levels for the qth attribute rank-ordered from the most preferable to the least preferable (the scale of the qth attribute}

   $$|X_q| = n_q \ (q \in K).$$
4. $X = X_1, \ldots, X_Q$ – a set of vectors $x_i \in X$ of the following type: $x_i = (x_{i1}, x_{i2}, \ldots, x_{iQ})$, where $x_{iq} \in X$.
5. $A = \{a_i\} \subseteq X$ – a subset of vectors describing the set of real alternatives.
6. An overall value of alternative $a_i \in A$ is evaluated using the formula

   $$V(a_i) = \sum_{q=1}^{Q} k_q v_q(x_{iq}), \qquad (1)$$

   where $a_i = (x_{i1}, x_{i2}, \ldots, x_{iQ})$, k_q is the non-negative weight of the qth attribute, and $v_q(x_{iq})$ is the value assigned to the attribute level x_{iq}.

By convention, we can choose the value of the best attribute level to equal 1, the value of the worst attribute level to equal 0, and we can normalize the attribute weights to sum to 1:

$$v_q(x_{1q}) = 1, \quad v_q(x_{n_q q}) = 0$$
$$(q = 1, 2, \ldots, Q), \text{ and } \sum_{q=1}^{Q} k_q = 1. \qquad (2)$$

Required: to select the preferred alternative of set A on the basis of decision maker preferences measured over multiple attributes.

There are two primary types of the information necessary:

- the relative importance (or preferability) of attributes used for evaluation of alternatives;
- the relative preferability (value) of separate levels upon attribute scales, assessed on alternatives.

The theoretically sound approaches to the determination of k_q and $v_q(x_{iq})$ for $i = 1, 2, \ldots, n$ and $q = 1, 2, \ldots, Q$ are based on the idea of drawing indifference curves (reflecting the indifference of a decision maker to gain some definite increment upon one or another attribute). This process is time consuming and it may be conceptually difficult for decision makers to be accurate even in the case of an additive value function of the form given in (1). (We note that this opinion of ours is not universally shared.)

If you look at practical tasks, described as applications of the decision aid methods (Corner and Kirkwood, 1991; Goodwin and Wright, 1991; Keeney, 1992), it can be seen that many if not all such tasks involve verbally described attribute levels assigned to alternatives. Even for many originally numerical attributes (such as area, time, distance, weight) in many cases it is necessary to construct discrete scales with verbal explanations (possibly including ranges of possible levels), to reflect the discrepancies in values. That is why many simplified approaches to weights and values evaluation have been developed. More details on these approaches can be found in Von Winterfeldt and Edwards (1986). However the situation is not the same for real decision tasks, in which we usually have to select from a group of alternatives close in value to the decision maker. Stillwell et al. (1981) pointed out that the situation changes dramatically when we exclude dominated alternatives (alternatives which have worse or equal performances over all attributes to some dominating alternative, and at least one inferior performance). In these circumstances slight differences in weights of attributes may lead to the reversals in ranking of decision alternatives (Larichev et al., 1993, 1995; Olson et al., 1995; Barron and Barrett, 1996). There are almost no analogous studies concerning assignment of values to the levels upon attribute scales, but it seems that the situation in this sphere is likely to be the same (see Olson et al., 1995; Moshkovich et al., 1998).

There are many experimental examples (see e.g. Schoemaker and Waid, 1982; Borcherding et al., 1991) which show that simple implementation of different techniques for deriving weights in the same task and with the same decision maker often lead to different results. In these circumstances specialists must be very careful in using simple multiattribute techniques for complicated tasks. In many cases this approach gives a very clear and simple result, but it's correctness is far from being obvious. Sensitivity analysis has been proposed for such cases (Von Winterfeldt and Edwards, 1986) to see if results change with varying model parameters. Mathematical programming and Monte Carlo simulation can be applied to generate weight sensitivity analysis, but with many dimensions, humans can sometimes have a difficult time sorting out the many contingent outcomes.

We can make several general conclusions from the material we have presented:

- in practical decision tasks most decisions involve qualitative attributes with no natural numerical equivalents (Larichev, 1992; Larichev and Moshkovich, 1997). Qualitative aspects can include such as the neighborhood or general quality of the building when purchasing a house, or color, interior, and ease of driving when purchasing an automobile;
- for qualitative as well as for originally quantitatively measured attributes it may be useful to define several distinct levels (maybe explained in words and examples) on a scale (Von Winterfeldt and Edwards, 1986; Goodwin and Wright, 1991; Keeney, 1992);
- levels on attribute scales as well as attribute importances may be rather easily and consistently rank ordered by a decision maker according to his (or her) preference (Watson and Buede, 1987; Larichev, 1992; Larichev and Moshkovich, 1997). Eckenrode's subjects stated that ranking was easier and more reliable than other methods (1965). Keeney (1992) stated that an ordinal scale is a logical first step for some attributes;
- carrying out theoretically based procedures for numerical estimation of these ranks is time consuming and cognitively uncomfortable for decision makers, poorly understood by them, leading to results that may be inadequate (Payne et al., 1988; Tversky et al., 1988);
- implementation of simplified procedures for numerical estimation of weights and values leads to unstable results, which may cause the wrong selection (Payne et al., 1993; Barron and Barrett, 1996);
- it is not always necessary to obtain full rank ordering of alternatives and thus implementation of numerical estimation is not always needed (Kirkwood and Sarin, 1985; Kirkwood and Corner, 1993; Barron and Barrett, 1996).

Implementation of ordinal judgments (ranking and ordinal comparison) is usually considered to be the most natural approach for decision makers and does not seem to involve problems to specialists in decision analysis (Payne et al., 1993). Ordinal comparisons are always the first practical step in preference elicitation procedures in multiattribute analysis. But almost always this step is followed by scaling procedures to obtain quantitative expressions for all elements of the model. There are ways to analyze the decision situation on the basis of ordinal judgments and sometimes to identify the preferred decision without resort to numbers (Kirkwood and Sarin, 1985; Kirkwood and Corner, 1993; Barron and Barrett, 1996). In the next section we discuss several models that use only ordinal, or partly ordinal, judgments for partial ranking of alternatives. To illustrate our ideas we use an example of an application problem analyzing 48 applicants for a tenure track position in MIS.

3. Methods to implement ordinal judgments for comparison of multiattribute alternatives

Possible types of ordinal preference information can be grouped as follows:

1. rank ordering of separate levels upon attribute scales (ordinal scales);
2. rank ordering of attributes upon their importance;
3. pairwise comparison of real alternatives;
4. ordinal tradeoffs: pairwise comparison of hypothetical alternatives differing in estimates of only two attributes.

3.1. Ordinal scales

The first category (ordinal attribute scales) is represented by one very easy and popular method, the rule of dominance. This rule states that one alternative is more preferable than another if it has attribute levels that are not less preferable on all attributes and is more preferable on at least one. This rule does not need the idea of attribute importance and is not necessarily connected with an additive form of a value function (although it does not apply to all functional forms). The rule of dominance may be easily applied as the first step in the analysis of the decision situation and may sometimes lead to a decision. But such a situation rarely occurs. Nevertheless, it is obvious that the best solution is contained in the set of non-dominated alternatives if the decision problem is completely modeled. Therefore, in practical tasks with large numbers of alternatives (which is often the case), dominance is a way to reduce the initial set for further analysis. The example using simple ordinal scales for multiattribute evaluation of applicants for a tenure track position in MIS is given in Table 1.

Here an alternative with ratings of average or better on all attributes would dominate an alternative with a rating of below average on one or more attributes, and average on all other attributes.

Table 1
Attributes with ordinal scale for applicants' evaluation

Attributes	Possible levels
A. Ability to teach our students	A1. Above average A2. Average A3. Below average
B. Ability to teach systems analysis and DBMS	B1. Above average B2. Average B3. Below average
C. Evaluation of completed research and scholarship	C1. Above average C2. Average C3. Below average
D. Potential in publications	D1. Above average D2. Average D3. Below average
E. Potential leadership in research	E1. Above average E2. Average E3. Below average
F. Match of research interests	F1. Above average F2. Average F3. Below average

3.2. Rank ordering of attributes upon importance

The next category (rank ordering of attributes upon importance) does not provide any decision rule by itself. In combination with ordinal scales and lexicographical attributes' ranking, the rule for comparison of alternatives may be as follows: first we select alternatives possessing the best possible level upon the most important attribute. From the resulting subset we select alternatives with the best possible level upon the next important attribute and so on. This rule is based on the assumption that *in the attributes' ranking one attribute is more important than all the other attributes, which follow it in the ranking*. This preemptive rule again does not necessarily imply the additive value function, but has the obvious drawback that because of its non-compensatory nature, this rule is rather unpopular theoretically.

If the ranking of attributes is supplemented by numeric alternatives' scores against separate attributes then there is a method proposed by Kirkwood and Sarin (1985). This method assumes that we know values for levels upon attribute scales, but we cannot derive exact weights, though we know the ranking of attributes upon importance, which allows to rank order weight coefficients in formula (1). They proved that if $k_1 > k_2 \cdots > k_Q$, then alternative **a** is more preferable, than alternative **b**, if

$$\sum_{q=1}^{p} v_q(a_q) - \sum_{q=1}^{p} v_q(b_q) \geqslant 0, \quad p = 1, 2, \ldots, Q, \qquad (3)$$

and at least one of these inequalities is strict (strictly more than 0).

This approach allows comparison of pairs of alternatives and use of this partial rank order in analyzing the task. The main advantage of this approach according to Kirkwood and Corner (1993) is that if the best alternative is identified by this procedure, it will be the best with any weights maintaining their rank order.

This approach is easy to use and it may be effective in some practical cases (see application described in Kirkwood and Sarin, 1985). Nevertheless, values need to be assigned to attribute levels. In addition the example application used in the article of Kirkwood and Sarin (1985) and simulations carried out for this procedure (Kirkwood and Corner, 1993) show that they use ordinal scales for attributes. To consider this rank as the value is very seldom true in real tasks.

3.3. Pairwise comparison of real alternatives

In some methods the decision maker is required to compare real alternatives at one or another step of the decision process (see, e.g. Korhonen, 1988). In general this information by itself will lead to the solution (if you compare all pairs of alternatives then you can construct a complete rank order of alternatives). But the whole area of multiattribute decision analysis has evolved from the notion that this task is too difficult for the decision maker. This approach is mostly used in multicriteria mathematical programming (in which there is not a finite number of alternatives for consideration). Still we consider this information as highly unstable (Tversky, 1969; Larichev, 1992).

3.4. Ordinal tradeoffs

The main idea of this approach is to ask the decision maker to make tradeoffs for each pair of attributes and for each pair of possible levels in the ordinal form. To carry out such a task we need to ask a decision maker questions of the kind:

> what do you prefer: to have this (better) level upon attribute q and that (inferior) level upon attribute $q+1$, or this (better) level upon attribute $q+1$ and that (inferior) level upon attribute q?

These questions may be asked for each pair of attributes and for each level upon attribute scales. It is clear, that the same question when levels are changed from the best to the worst attribute level, correspond to the questions in the "swing" procedure for attribute weights (Von Winterfeldt and Edwards, 1986; Edwards and Barron, 1994), but does not require quantitative estimation of the preference.

The same information may be obtained with far fewer questions by comparing two hypothetical alternatives that differ in performance on only two attributes. This type of information is considered to be rather reliable (and can be checked for transitivity). This type of information is partially implemented in method ZAPROS (Larichev and Moshkovich, 1995, 1997). In general this information allows us to use the following rule for comparison of two alternatives:

> alternative **a** is not less preferable than alternative **b**, if for each pair of estimates (a_i, a_j) of alternative **a** there exists a not more preferable pair of estimates (b_k, b_l) of alternative **b**.

The above analysis shows that the most promising types of ordinal information that can be used for comparison of multiattribute alternatives may be presented by two forms: ordinal scales for attributes and ordinal tradeoffs. Let us analyze the appropriate procedures and results provided by these two types of preference information.

4. Three possible steps in using ordinal preference judgments

4.1. Dominance rule

The first step in any decision analysis is to form the set of alternatives, form the set of attributes, and evaluate alternatives against attributes. If we decide to use ordinal judgments for comparison of alternatives, the first step in this direction is to elaborate ordinal scales for attributes. This information allows pairwise comparison of real alternatives according to the following rule (see Fig. 1):

> alternative **a** is not less preferable than alternative **b**, if for each attribute q $(q = 1, 2, \ldots, Q)$ estimate a_q of alternative **a** is not less preferable than estimate b_q of alternative **b**.

We presented an example of six attributes used to evaluate applicants for a tenure track position

	Alternatives		
Attributes	a	$\succ$	B
1	a_1	→	b_1
.	.		.
.	.		.
i	a_i	→	b_i
.	.		.
.	.		.
Q	a_Q	→	b_Q

Fig. 1. Comparison of real alternatives upon dominance.

in MIS in Table 1. The ordinal scales in this case were the same for all attributes using levels: above average, average, and below average. It can be argued that a small number of categories capture the essence of a decision maker's scale of value for an attribute. This has the disadvantage of converting a continuous scale to a discrete scale, but with the compensating advantage of being more manageable.

4.2. Joint ordinal scale (JOS)

In the ZAPROS method (Larichev and Moshkovich, 1995, 1997) partial information on ordinal tradeoffs is used. We describe the procedure in a more detail.

The decision maker is asked to compare pairs of hypothetical alternatives, each with the best levels of attainment on all attributes but one. Thus, the decision maker is asked to compare pairs of hypothetical alternatives from the list $L \subset X$,

$$L = \{x_i \in X \mid x_{iq} = x_{1q} \ \forall q \in K, \\ \text{except one } t \text{ such that } x_{it} \neq x_{1t}\}.$$

The number of these alternatives is not large: $N = \sum_{q=1}^{Q}(n_q - 1) + 1$.

Thus, the decision maker is to compare alternatives x_i and x_j, differing in attainment levels on only two attributes, holding all other attributes values at the same level. Possible responses are: (1) x_i is preferred to x_j; (2) x_i and x_j are equally preferable; and (3) x_j is preferred to x_i. Such a comparison is demonstrated for the example in Fig. 2.

The resulting ranking of alternatives from the set L (see the last column in Table 2) forms the JOS (Larichev and Moshkovich, 1995, 1997). JOS index $J(x_{iq})$ shows the rank of corresponding attribute level among all possible attribute values (the smaller the index the better the corresponding

Attributes	Alternative 1		Alternative 2	
A. Ability to teach our students	Above Average	A1	Above Average	A1
B. Ability to teach systems analysis and DBMS	Above Average	B1	Above Average	B1
C. Evaluation of completed research and scholarship	Above Average	C1	Above Average	C1
D. Potential in publications	**Average**	**D2**	**Above Average**	**D1**
E. Potential leadership in research	Above Average	E1	Above Average	E1
F. Match of research interests	**Above Average**	**F1**	**Below Average**	**F3**

POSSIBLE ANSWERS:

1. Alternative 1 is more preferable, than alternative 2.
2. Alternatives 1 and 2 are equally preferable.
3. Alternative 2 is more preferable, than alternative 1.

Fig. 2. Comparison of hypothetical alternatives for construction of JOS.

attribute level). Note, that $J(x_{11}) = J(x_{12}) = \cdots = J(x_{1Q}) = 1$. Thus, we construct a unique ordinal scale for all attributes with their possible values. A JOS for the example is given in Table 2.

Construction of the JOS provides a simple rule for comparison of multiattribute alternatives. Each vector $\mathbf{a} = (a_1, a_2, \ldots, a_Q)$ may be rewritten in the form of *index vector* $J(\mathbf{a}) = (J(a_1), J(a_2), \ldots, J(a_Q))$, in which each component is substituted with its JOS index. The advantage of this presentation is due to *the comparability of JOS indices among attributes* (we are not able to compare x_{iq} and x_{jt}, but the JOS index $J(x_{iq})$ is always comparable with the JOS index $J(x_{jt})$). Then the rule for comparison of the two alternatives on the basis of the JOS is the following:

> alternative **a** is not less preferable than alternative **b**, if for each component a_i of alternative **a** there may be found component b_j of alternative **b** such that $J(a_i) \leqslant J(b_j)$.

The correctness of the rule in case of an additive value function was proven in Larichev and Moshkovich (1995). If we rearrange elements of each index vector in an ascending order (as all indices are comparable), we can say that:

> alternative **a** is not less preferable than alternative **b** if for each $q = 1, 2, \ldots, Q$ $J_q(a) \leqslant J_q$ (**b**).

Thus, estimating **a** and **b** through the JOS leads us to the simple rule of dominance for comparison of two alternatives presented in the form of their JOS indices (see Fig. 3(a and b)). As a result by acquiring information on comparison of alternatives from L we are able to construct a new presentation of alternatives (by their JOS indices), rearrange them in an ascending order, and then use the rule of dominance to compare real alternatives.

The assumed transitivity of preferences and rank orderings of attribute levels make it possible to construct an effective procedure of pairwise comparisons (for more details on the procedure see Larichev and Moshkovich, 1995, 1997).

Using this rule it is possible to compare some pairs of real alternatives but it does not guarantee the comparison of all pairs of alternatives

Table 2
Joint ordinal scale for the applicant selection task

Attribute	Attribute values	JOS index	Corresponding vector from L
A	A1	J(A1) = J(B1) = J(C1) = 1	A1, B1, C1, D1, E1, F1
B	B1	J(D1) = J(E1) = J(F1) = 1	
C	C1		
D	D1		
E	E1		
F	F1		
C	C2	J(C2) = J(E2) = 2	A1, B1, C2, D1, E1, F1
E	E2		A1, B1, C1, D1, E2, F1
A	A2	J(A2) = J(D2) = J(F2) = 3	A2, B1, C1, D1, E1, F1
D	D2		A1, B1, C1, D2, E1, F1
F	F2		A1, B1, C1, D1, E1, F2
B	B2	J(B2) = 4	A1, B2, C1, D1, E1, F1
B	B3	J(B3) = J(E3) = J(F3) = 5	A1, B3, C1, D1, E1, F1
E	E3		A1, B1, C1, D1, E3, F1
F	F3		A1, B1, C1, D1, E1, F3
A	A3	J(A3) = J(C3) = J(D3) = 6	A3, B1, C1, D1, E1, F1
C	C3		A1, B1, C3, D1, E1, F1
D	D3		A1, B1, C1, D3, E1, F1

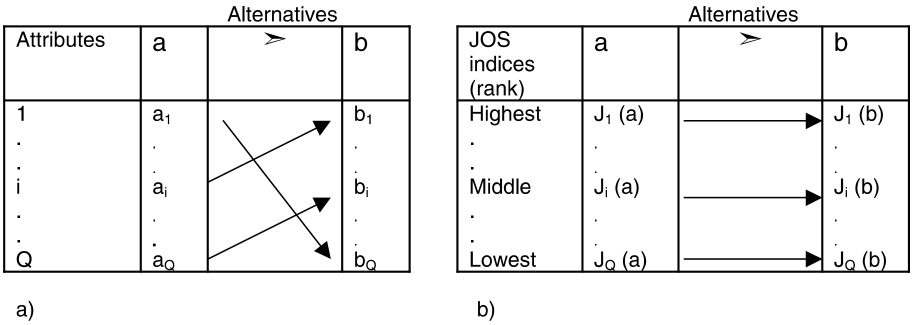

Fig. 3. Comparison of real alternatives upon JOS: $J_1(x) < J_2(x) < \cdots < J_Q(x)$: (a) actual information; (b) using rearranged in ascending order JOS indices.

(Moshkovich et al., 1998). We will demonstrate these concepts using the example given in that paper, to select faculty applicants to visit a campus for an interview. This process involves selection of about three candidates to bring to campus (at some expense) out of as many as 50 applicants for a faculty position. For that example, the upper part of the partial rank order of subjects in the example is presented in Fig. 4. Applicant 26 is the only one with rank 1. Applicant 45 is the only one with rank 2. There are five applicants with the rank of 3.

4.3. Paired joint ordinal scale (PJOS)

Our approach to preference elicitation is based on ordinal comparison of two alternatives that differ in values (or components) by only *two attributes* while all other attribute values are the same for both alternatives. Note that in the presented procedure for ordinal tradeoffs used in ZAPROS, only a small part of the admissible comparisons are used to form the JOS – elements from the subset L. Thus, it is possible to ask the decision maker to compare alternatives with *different levels* on only two attributes, but not necessarily including the *best* level as in the vectors from L.

Let x_i and x_j be two alternatives, which have the best components accept attributes p and q. If the decision maker prefers x_i to x_j, it means that combination of estimated (x_{iq}, x_{ip}) is preferred to

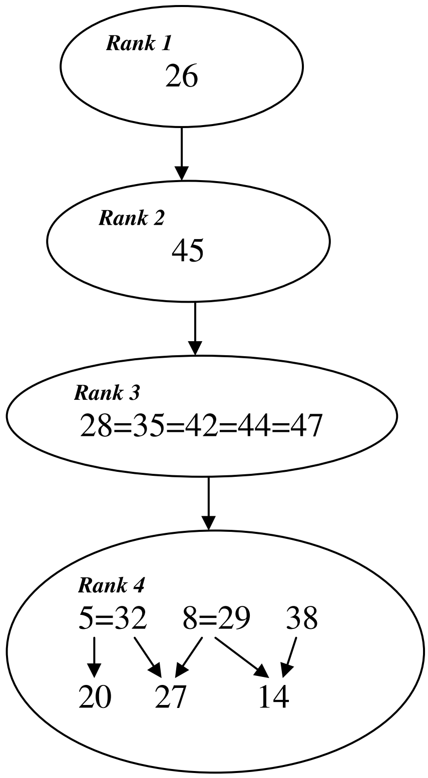

Fig. 4. Upper part of partial rank ordering of applicants on the basis of JOS.

(x_{jq}, x_{jp}), as all other components in index vectors are equal to 1 (see an example of the comparison for the application problem in Fig. 5). Thus, such comparison may be viewed as a comparison of *pairs of estimates against different attributes*.

Attributes	Alternative 1		Alternative 2	
A. Ability to teach our students	Above Average	A1	Above Average	A1
B. Ability to teach systems analysis and DBMS	Above Average	B1	Above Average	B1
C. Evaluation of completed research and scholarship	Above Average	C1	Above Average	C1
D. Potential in publications	**Average**	D2	**Above Average**	D1
E. Potential leadership in research	Above Average	E1	Above Average	E1
F. Match of research interests	**Average**	F2	**Below Average**	F3

POSSIBLE ANSWERS:

1. Alternative 1 is more preferable, than alternative 2.
2. Alternatives 1 and 2 are equally preferable.
3. Alternative 2 is more preferable, than alternative 1.

Fig. 5. Eliciting preferential information on comparison of pairs of ranks (33 vs. 15).

Let us assume that we know the relationship between all possible pairs of attribute estimates. Then we are able to construct the PJOS and use it for comparison of real alternatives. PJOS index $PJ(x_{iq}, x_{ip})$ shows the rank of corresponding combination of two attribute levels among all possible attributes' values (the smaller the index the better the corresponding attribute level).

Construction of the PJOS provides the following rule for comparison of multiattribute alternatives. Each vector $\mathbf{a} = (a_1, a_2, \ldots, a_Q)$ may be rewritten in the form of *paired vector* $PJ(\mathbf{a})$, in which pairs of components are substituted with their PJOS index. Then the rule for comparison of the two alternatives on the basis of the PJOS is the following:

> alternative **a** is not less preferable than alternative **b**, if for each pair of estimates (a_i, a_j) of alternative **a** there exists a pair of estimates (b_k, b_l) of alternative **b** such that $PJ(a_i, a_j) \leqslant PJ(b_k, b_l)$.

The proof of the correctness of the rule in case of additive value function is given in the Appendix A. If we rearrange elements of each paired vector in an ascending order (as all indices are comparable), we can say that:

> alternative **a** is not less preferable than alternative **b** if for each $k = 1, 2, \ldots Q/2$ (half of the previous number of attributes) $PJ_k\ (\mathbf{a}) \leqslant PJ_k(\mathbf{b})$.

Thus, estimating **a** and **b** through the PJOS leads us to the simple rule of dominance for comparison of two alternatives presented in the form of their PJOS indices (see Fig. 6(a and b)). As a result of acquiring such information we are able to construct a new presentation of alternatives (by their PJOS indices), rearrange them in an ascending order, and then use the rule of dominance to compare real alternatives.

Although the logic of this presentation is theoretically the same as with JOS, there are three problems connected with the construction and implementation of PJOS:

1. the number of required comparisons for the construction of PJOS is very large (exceeds the reasonable time the decision maker can spare for such a task by a great deal);

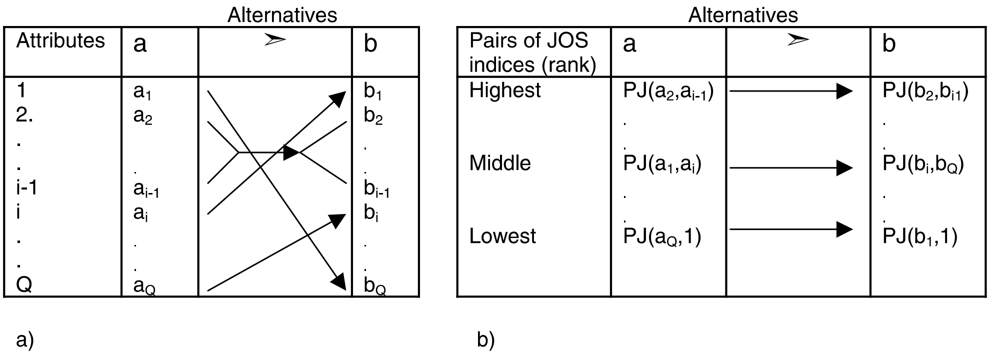

Fig. 6. Comparison of alternatives upon Paired JOS (PJOS): $PJ_1(x,x) < PJ_2(x,x) < \cdots < PJ_m(x,x)$: (a) actual information; (b) using rearranged in ascending order PJOS indices.

2. not all pairs of attribute estimates can be compared in PJOS (as they may require comparison of hypothetical alternatives differing in estimates against more than two attributes: e.g., 3 or 4);
3. it is possible to divide alternative estimates into different pairs (there is a variability in the presentation of an alternative through PJOS indices).

Thus, it is not efficient to try to obtain *full* information. As the number of real alternatives that need to be compared has been reduced after implementation of the JOS, a reasonable approach would be to use an iterative process to effectively identify the needed discriminatory power while not requiring exhaustive numbers of comparisons (see Olson, 1996).

It is possible to use previous information to reach some conclusions about the comparison of pairs of attribute estimates on the basis of JOS indices. This is based on transitivity of the constructed preference relation.

According to the construction of the JOS, if $J(x_{ls}) \leqslant J(x_{iq})$ and $J(x_{mt}) \leqslant J(x_{jp})$, then $(J(x_{ls}), J(x_{mt}))$ is preferred to (or is equally preferable to) $(J(x_{iq}), J(x_{jp}))$. On the basis of the transitivity of the constructed binary relation it is possible to conclude that, If $(J(x_{iq}), J(x_{jp}))$ is preferred to $(J(x_{ls}), J(x_{mt}))$ then:

(1) $\forall J(x_{kf}) \leqslant J(x_{\mathrm{iq}})$ and $J(x_{dg}) \leqslant J(x_{jp})$: $(J(x_{kf}), J(x_{dg}))$ is preferred to $(J(x_{ls}), J(x_{mt}))$;
(2) $\forall J(x_{kf}) \geqslant J(x_{ls})$ and $J(x_{dg}) \geqslant J(x_{mt})$: $(J(x_{ls}), J(x_{mt}))$ is preferred to $(J(x_{kf}), J(x_{dg}))$.

These properties allow construction of an effective iterative procedure (Olson, 1996) making it possible to check the consistency of some of the decision maker's responses (on the basis of transitivity):

1. Select two real alternatives **a** and **b** not comparable upon the JOS. Form their JOS representation in ascending order $J(a)$ and $J(b)$. Delete all JOS indices equal in both alternatives. Let us consider that we have m indices left: $J_1(a) \leqslant J_2(a) \leqslant \ldots \leqslant J_m(a)$ and $J_1(b) \leqslant J_2(b) \leqslant \cdots \leqslant J_m(b)$. For the each pair of corresponding indices mark 1 if $J_i(a) \leqslant J_i(b)$ and 0 otherwise (see Fig. 7).
2. Form all possible pairs of indices for comparison that can represent these alternatives: combine 1 and 0 in each pair. The number of possible pairs will be equal to the number of 1s multiplied by the number of 0s (see Fig. 7).
3. For formed pairs of JOS indices check if it can be described by estimates of only two attributes (there is a variability here as the same JOS index may be assigned to several different attribute levels – see Table 2).
4. Form hypothetical alternatives differing in estimates on two attributes which represent comparison of these pairs of JOS indices. Use the result to compare real alternatives.

In the example, first, incomparable alternatives #5 = (A1, B1, C1, D2, E2, F2) and #38 = (A1, B3, C1, D1, E1, F1) are analyzed. According to the JOS presented in the Table 2, J(#5) = (1, 1, 1, 3, 2, 3) and J(#38) = (1, 5, 1, 1, 1, 1). We rearrange in

Alternatives		Direction of preference (1 - more, 0 - less)
a	**b**	
$J_1(a)$	$J_1(b)$	1
$J_2(a)$	$J_2(b)$	0
.	.	.
.	.	.
$J_m(a)$	$J_m(b)$	0

Possible pairs of JOS indices for comparison:

$[J_1(a),J_2(a)]$ vs. $[J_1(b),J_2(b)]$

$[J_1(a),J_m(a)]$ vs. $[J_1(b),J_m(b)]$

Fig. 7. Formation of pairs of JOS indices for comparison.

ascending order following the ZAPROS procedure. The resulting index vectors: $J(\#5) = (1,1,1,2,3,3)$ and $J(\#38) = (1,1,1,1,1,5)$. According to the proposed procedure:

(1) Eliminate all equal ranks and mark 1 and 0 for more preferable and less preferable pairs of ranks with respect to the first alternative (#5):

Alternatives	#5	#38	Mark
	2	1	0 (less preferable)
	3	1	0 (less preferable)
	3	5	1 (more preferable)

(2) Form all possible pairs (combinations of 1s and 0s). There are only two possible variants in our case:

23 vs. 15 or 33 vs. 15.

(3) Check if these pairs can be presented by hypothetical alternatives differing in estimates against only two attributes. To do this we first form possible values for each of the JOS indices:

1 can be presented by any of the following {A1, B1, C1, D1, E1, F1};

5 can be presented by any of the following {B3, E3, F3};

2 can be presented by any of the following {C2, E2};

3 can be presented by any of the following {A2, D2, F2}.

The analysis shows that each of the two variants is admissible.

(4) Form hypothetical alternatives for comparison. First we form hypothetical alternatives to compare 33 vs.15. Corresponding alternatives are (see Fig. 5): (A1, B1, C1, D2, E1, F2) and (A1, B1, C1, D1, E1, F3).

The second alternative was preferred by the decision maker. This allows us to conclude that Alternative #38 is preferred to alternative #5.

We identify the JOS index vectors for alternatives #5 = (A1, B1, C1, D2, E2, F2) and #8 = (A1, B2, C2, D1, E1, F1). Corresponding JOS index vectors are: $J(\#5) = (1,1,1,2,3,3)$ and $J(\#8) = (1,1,1,1,2,4)$.

(1) Eliminate equal ranks and put corresponding 1 and 0 marks:

Alternatives	#5	#8	Mark
	3	1	0 (less preferable)
	3	4	1 (more preferable)

(2) Form possible pairs of indices. There is only one possible pair: 33 vs. 14.

It was previously stated that combination of ranks 15 is preferable to combination of ranks 33. Using transitivity we are able to conclude that 14 is preferred to 33, and therefore alternative #8 is preferred to alternative #5. This procedure does not guarantee the comparison of any two real alternatives, but may add the necessary information for the solution of the problem.

5. Effectiveness of the information on ordinal tradeoffs

The approach presented above shows how we are able to use partial information about the decision makers preferences. The crucial point of the approach is the use of ordinal judgments for all elements in the decision analysis. We use ordinal scales for attributes, we use ordinal information on relative importance of different attributes, and different attribute values.

In the process we ask the decision maker to compare pairs of hypothetical alternatives as follows:

1. alternatives differing in values against one attribute (to construct ordinal scales for attributes),
2. alternatives differing in values against two attributes while each alternative has only one attribute value different from the best one (to construct JOS), and
3. alternatives differing in values against two attributes while each alternative can have up to two value different from the best ones (to construct PJOS).

Each type of information may be used to compare some pairs of *real* alternatives. Constructed ordinal scales may be used to compare real alternatives on the basis of the dominance rule. The Constructed JOS can add additional comparisons among real alternatives (by presenting them with JOS indices and using them for comparison of alternatives on the basis of dominance relation). Constructed PJOS can still produce some additional comparisons among real alternatives (by presenting alternatives through PJOS indices, and use, once again, dominance for comparison).

In general we can assume that these three steps in acquiring additional information on the decision maker's preferences can give the solution to the problem, but what are the chances for that? How effective are these procedures?

To answer these questions simulation was carried out to evaluate the percentage of pairs among real alternatives compared at each step of obtaining information on decision maker's preferences. The larger the proportion of compared alternatives, the closer the constructed partial order will be to the complete ranking. The number of comparisons of hypothetical alternatives required from the decision maker was calculated.

The program was written using electronic spreadsheets with VBA. The simulation assumed one decision maker with a definite value function. The decision maker was modeled through an additive value function (which was used to get information on comparison of hypothetical alternatives). The program simulated the process of constructing JOS and carrying out additional judgments for pairs of JOS indices (as stated in the previous sections).

The value function stayed the same (one decision maker), but the set of real alternatives was formed each round of simulation. Attribute value (an integer, reflecting the place of the value in the ordinal scale of the attribute) for each alternative was formed by using the built-in random number generator, based on uniform distribution.

Simulation was carried out for different numbers of attributes (5 and 7), numbers of possible attribute levels (3 and 5), and numbers of alternatives in the initial set (30 and 50). (The method is more appropriate for multiattribute problems with more than 10 alternatives.) These parameters are considered to reflect the task environment for the proposed approach: (1) alternatives are evaluated through ordinal scales, (2) preference structure is constructed in the attribute space (not in the space limited by the presented set of real alternatives), and (3) the task is to rank order of alternatives.

Combination of two possible values for each of three parameters (number of alternatives, number of attribute levels, and number of attributes) produces eight possible combinations. For each variant the process was simulated 500 times (each time the new initial set of real alternatives was generated).

For each of 500 simulations, the percent of compared pairs in the set of real alternatives was calculated, and the number of comparisons carried out by "the decision maker" while constructing PJOS was marked (two other steps require a stable number of comparisons for any set of real alternatives). For both indices and for each combination, three parameters were calculated to summarize the result over 500 trials: average, standard deviation, and maximum/minimum values. Averages are presented in Table 3.

The results show that on the average as many as up to 75–80% of the alternatives can be compared using only ordinal judgments in the presented form (see Table 1). Although the proportional number of compared pairs of alternatives decreases with the growth of the number of attributes and attribute levels, it is almost the same for different numbers of alternatives (see Fig. 8(a) and (b)). The difference between cases for 30 and 50 alternatives is less than 1% for the first two steps (Dominance and JOS) and it's rises to 3% at the third step (PJOS).

At the same time the number of alternatives influences the number of questions to the decision maker for the additional comparison of alternatives (it becomes much larger with the growth of the number of alternatives).

Growth in the number of attributes decreases the percent of compared alternatives, while the number of attribute levels influences the number of additional questions to the decision maker.

The conclusions in this part are based mainly on the averages obtained. To give an idea of how diversified the initial data was, we provide information on standard deviations (Table 4) and ranges of data (in Table 5).

Table 3
Average percentage of compared alternatives

	Number of attributes = 5				Number of attributes = 7			
Number of levels	3		5		3		5	
Number of alternatives	30	50	30	50	30	50	30	50
Method								
Dominance	26.4%	26.1%	16.1%	15.2%	11.5%	11.5%	5.9%	5.2%
JOS	65.1%	65.0%	50.3%	49.3%	54.3%	54.4%	37.8%	37.8%
PJOS (pairs of indices)	75.6%	76.1%	72.9%	74.2%	62.7%	63.7%	56.4%	59.0%
Number of additional questions (PJOS)	14	17	63	96	21	30	86	141

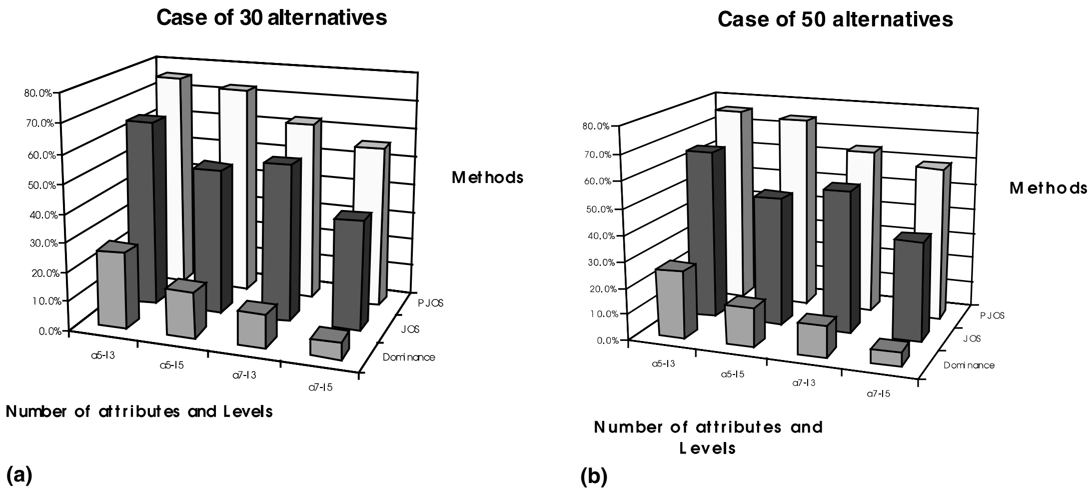

Fig. 8. Results of simulation (percentage of compared alternatives).

Table 4
Standard deviations for the parameters

	Number of attributes = 5				Number of attributes = 7			
Number of levels	3		5		3		5	
Number of alternatives	30	50	30	50	30	50	30	50
Method								
Dominance	5.2%	4.2%	4.3%	3.3%	3.4%	2.6%	2.8%	1.8%
JOS	6.1%	4.5%	6.4%	4.9%	6.6%	5.3%	6.6%	4.7%
PJOS (pairs of indices)	4.9%	3.6%	4.8%	3.4%	6.0%	4.7%	5.8%	4.1%
Number of additional questions (PJOS)	2.2	2.2	8.3	10.0	3.8	4.3	10.4	11.8

Table 5
Maximum (minimum) values for the parameters

	Number of attributes = 5				Number of attributes = 7			
Number of levels	3		5		3		5	
Number of alternatives	30	50	30	50	30	50	30	50
Method								
Dominance	40 (14)%	37 (17)%	26 (5)%	27 (8)%	21 (5)%	19 (6)%	17 (2)%	11 (2)%
JOS	82 (51)%	76 (53)%	64 (35)%	60 (38)%	73 (37)%	64 (41)%	59 (23)%	52 (23)%
PJOS (pairs of indices)	89 (64)%	86 (68)%	83 (62)%	82 (66)%	78 (47)%	72 (52)%	72 (39)%	71 (46)%
Number of additional questions (PJOS)	19 (8)	22 (13)	80 (46)	127 (71)	30 (12)	44 (20)	113 (58)	165 (114)

The data shows enough stability in the values to assume that the averages represent the data sets adequately. Although we cannot guarantee the choice of the best alternative we are able to provide a rather narrow subset of better alternatives for further analysis. In accordance with the framework of using ordinal information for comparison of alternatives, the general suggestion (if one best alternative cannot be selected), to modify the description of the selected alternatives (change attributes and scales) to better present differences in them (see, e.g. Larichev and Moshkovich, 1997).

6. Conclusion

The process of information elicitation from decision makers and experts is a necessary element in multiattribute analysis. This information is required for elimination of the uncertainty connected with the presence of multiple criteria, to elaborate the necessary compromises, and to identify good decisions. It is preferable to use human judgments in a qualitative form: it is more natural for people and provides more reliable information. Qualitative information can be used in a logical and theoretically correct manner for comparing and evaluating multiattribute alternatives.

Quantitative measurement of qualitative notions may lead to an incorrect result, which is difficult to detect. In this case the impression is given that the appropriate decision has been identified, as we substitute decisions of a consultant (or the author of the method) for the real decision maker. In general, non-essential differences in numerical expression of values and weights may result in invalid application of that methodology to decision makers.

In many practical cases we are able to identify a preferred decision without resort to numerical scaling. We have shown that ordinal information can be effective in comparison of multiattribute alternatives. Obtaining ordinal information can be a logical first step in solving almost any problem. In this case the described procedures of ordinal tradeoffs may lead to a satisfactory, easily explainable and reliable solution. Such forms of human judgment allow conducting logical analysis of the elicited preferential information, detection of possible inconsistencies, and overcoming these through additional analysis.

Appendix A. Proof of the rule

We have that for each $q, p \in K$ $\exists t(q)$, $t(p)$ such that (a_{iq}, a_{ip}) is preferable to or is equally preferable to $(a_{jt(q)}, a_{jt(p)})$. This means that

$$k_q v_q(a_{iq}) + k_p v_p(a_{ip}) \geqslant k_{t(q)} v_{t(q)}(a_{jt(q)}) + k_{t(p)} v_{t(p)}(a_{jt(p)}).$$

Summing these inequalities for each pair of estimates, we obtain

$$\sum_{q=1}^{Q} k_q v_q(a_{iq}) \geqslant \sum_{q=1}^{Q} k_q v_q(a_{jq}).$$

Thus $v(a_i) \geqslant (va_j)$, and a_i is preferable to or is equally preferable to a_j.

References

Barron, F.H., 1973. Using Fishburn's techniques for analysis of decision trees: Some examples. Decision Sciences 4 (2), 247–267.

Barron, F.H., 1992. Selecting a best multiattribute alternative with partial information about attribute weights. Acta Psychologica 80, 91–103.

Barron, F.H., Barrett, B.E., 1996. Decision quality using ranked attribute weights. Management Science 42 (11), 1515–1523.

Brans, J.P., Vincke, P.H., 1985. A preference ranking organization method, the PROMETHEE method. Management Science 31 (6), 647–656.

Borcherding, K., Eppel, T., Von Winterfeldt, D., 1991. Comparison of weighting judgments in multiattribute utility measurement. Management Science 37 (2), 1603–1619.

Cook, W.D., Kress, M., 1992. Ordinal Information and Preference Structure (Decision Models and Applications). Prentice-Hall, Englewood Cliffs, NJ.

Corner, J.L., Kirkwood, C.W., 1991. Decision analysis applications in the operations research literature, 1970–1989. Operations Research 39, 206–219.

Edwards, W., Barron, F.H., 1994. Smarts and smarter: Improved simple methods for multiattribute utility measurement. Organizational Behavior and Human Decision Processes 60, 306–325.

Fishburn, P.C., 1964. Decision and Value Theory. Wiley, New York.

Goodwin, P., Wright, G., 1991. Decision Analysis for Management Judgments. Wiley, New York.

Kahneman, D., Slovic, P., Tversky, A. (Eds.), 1982. Judgment under Uncertainty: Heuristics and Biases. Cambridge University Press, Cambridge.

Keeney, R., 1992. Value-Focused Thinking. Harvard University Press, Cambridge, MA.

Keeney, R., Raiffa, H., 1976. Decisions with Multiple Objectives: Preference and Value Tradeoffs. Wiley, New York.

Kirkwood, C.W., Corner, J.L., 1993. The effectiveness of partial information about attribute weights for ranking alternatives in multiattribute decision making. Organizational Behavior and Human Decision Processes 54, 456–476.

Kirkwood, C.W., Sarin, R.K., 1985. Ranking with partial information: A method and an application. Operations Research 33, 38–48.

Korhonen, P., 1988. A visual reference direction approach to solving discrete multiple criteria problems. European Journal of Operational Research 34 (2), 152–159.

Korhonen, P., Wallenius, J., Zionts, S., 1984. Solving the discrete multiple criteria problem using convex cones. Management Science 30 (11), 1336–1345.

Larichev, O.I., 1992. Cognitive validity in design of decision-aiding techniques. Journal of Multi-Criteria Decision Analysis 1, 127–138.

Larichev, O.I., Moshkovich, H.M., 1995. ZAPROS-LM – A method and system for ordering multiattribute alternatives. European Journal of Operational Research 82, 503–521.

Larichev, O.I., Moshkovich, H.M., 1997. Verbal Decision Analysis for Unstructured Problems. Kluwer Academic Publishers, Berlin.

Larichev, O.I., Moshkovich, H.M., Mechitov, A.I., Olson, D.L., 1993. Experiments comparing qualitative approaches to rank ordering of multiattribute alternatives. Journal of Multi-Criteria Decision Analysis 2 (1), 5–26.

Larichev, O.I., Olson, D.L., Moshkovich, H.M., Mechitov, A.I., 1995. Numeric vs. cardinal measurements in multiattribute decision making (How exact is enough?). Organizational Behavior and Human Decision Processes 64 (1), 9–21.

Lofti, V., Stewart, T.J., Zionts, S., 1992. An aspiration-level interactive model for multiple criteria decision making. Computers and Operations Research 19 (7), 671–681.

Moshkovich, H.M., Schellenberger, R., Olson, D.L., 1998. Data influences the result more than preferences: Some lessons from implementation of multiattribute techniques in a real decision task. Decision Support Systems 22, 73–84.

Olson, D.L., 1992. Review of empirical studies in multiobjective mathematical programming: Subject learning and response to nonlinear utility. Decision Sciences 23 (1), 1–20.

Olson, D.L., 1996. Decision Aids for Selection Problems. Springer, New York.

Olson, D.L., Moshkovich, H.M., Schellenberger, R., Mechitov, A.I., 1995. Consistency and accuracy in decision aids: Experiments with four multiattribute systems. Decision Sciences 26, 723–748.

Payne, J.W., Bettman, J.R., Johnson, E.J., 1988. Adaptive strategy selection in decision making. Journal of Experimental Psychology: Learning, Memory, and Cognition 14, 534–552.

Payne, J.W., Bettman, J.R., Johnson, E.J., 1993. The Adaptive Decision Maker. Cambridge University Press, New York.

Roy, B., 1968. Classement et choix en presence de critères multiples. RIRO 8, 57–75.

Salminen, P., Korhonen, P., Wallenius, J., 1989. Testing the form of a decision maker's multiattribute value function based on pairwise preferences. Journal of the Operational Research Society 40 (1), 101–104.

Schoemaker, P.J.H., Waid, C.C., 1982. An experimental comparison of different approaches to determining weights in additive utility models. Management Science 12 (2), 182–196.

Slovic, P., Fishhoff, B., Lichtenstein, S., 1977. Behavioral decision theory. Annual Psychological Review 28.

Stillwell, W.G., Seaver, D.A., Edwards, E., 1981. A comparison of weight approximation techniques in multiattribute utility decision making. Organizational Behavior and Human Decision Processes 28, 62–77.

Steuer, R.E., 1986. Multiple Criteria Optimization: Theory, Computation, and Application. Wiley, New York.

Tversky, A., 1969. Intransitivity of preferences. Psychological Review 76, 31–48.

Tversky, A., Sattath, S., Slovic, P., 1988. Contingent weighting in judgment and choice. Psychological Review 95, 371–384.

Von Winterfeldt, D., Edwards, W., 1986. Decision Analysis and Behavioral Research. Cambridge University Press, Cambridge.

Watson, S.R., Buede, D.M., 1987. Decision Synthesis. Cambridge University Press, Cambridge.

Wierzbicki, A.P., 1982. A mathematical basis for satisfying decision making. Mathematical Modelling 3, 391–405.