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Theory and Methodology

# Comparison of the REMBRANDT system with analytic hierarchy process

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**Abstract**

This study compared the use of the REMBRANDT system to analytic hierarchy process (AHP) in a group multiple criteria selection problem. The decision was to select an operations management text, and the decision was the responsibility of a faculty group. The primary purpose of this study is to apply REMBRANDT to the same problem that was supported by AHP. There are no noticeable differences between the techniques from the perspective of the users, as the same input is used. Important technical differences between AHP and REMBRANDT are demonstrated, including (1) different ratio input scales, (2) alternative calculation of impact scores, and (3) a different aggregation procedure. REMBRANDT was found to recommend the same decision as AHP when the geometric mean was used for aggregation, but a different decision was given by conventional AHP using arithmetic mean aggregation.

*Keywords:* Multiple criteria decision making; Analytic hierarchy process (AHP); Group decision support systems

## 1. Introduction

The analytic hierarchy process (Saaty, 1977) has proven to be a very popular technique for aiding multiple criteria selection problems (Zahedi, 1986; Shim, 1989). However, there have been a number of criticisms of the technique, including identified problems of rank reversal (Belton and Gear, 1983, 1985; Schoner and Wedley, 1989; Schoner, Wedley and Choo, 1990), aggregation, and scale (Lootsma, 1991).

A group in the Netherlands, led by F.A. Lootsma, has developed a system which uses **R**atio **E**stimation in **M**agnitudes or deci-**B**ells to **R**ate **A**lternatives which are **N**on-**D**ominaTed (Lootsma, Mensch and Vos, 1990; Lootsma, 1992). This system is intended to adjust for three contended flaws in AHP. First, direct rating is on a logarithmic scale (Lootsma, 1988), which replaces the fundamental 1–9 scale presented by Saaty. Second, the Perron-Frobenius eigenvector method of calculating weights is

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replaced by the geometric mean, which avoids potential rank reversal (Barzilai, Cook and Golanyi, 1987). And third, aggregation of scores by arithmetic mean is replaced by the product of alternative relative scores weighted by the power of weights obtained from analysis of hierarchical elements above the alternatives.

The purpose of this paper is to comparatively examine the REMBRANDT system with AHP on a group decision selection problem involving multiple criteria. The decision problem involved selection of an operations management textbook by a faculty committee. The group decision was supported by the actual use of AHP. The data generated in that study is used in applying REMBRANDT to the same problem. Computational features of both systems are compared, and group aspects of both systems are analyzed with the intent of identifying positive features of both methods.

The second section of this paper discusses REMBRANDT, following the features of AHP which were designed to be improved. This section is considered necessary because REMBRANDT is a new technique with publication pending (Lootsma, 1992). The third section briefly describes the group decision problem, and includes the AHP results. The fourth section presents the REMBRANDT results, using software courteously provided by Professor Lootsma. The fifth section analyzes differences, especially relating to group decision support aspects. The sixth section presents conclusions.

## 2. The REMBRANDT system

The REMBRANDT system has been designed to address three criticized features of AHP. The first issue addressed by Lootsma is the numerical scale for verbal comparative judgment. Saaty presented a verbal scale for the ratio of relative value between two objects where 1 represents roughly equal value, 3 represents the base object as being moderately more important than the other object, 5 reflects essential advantage, 7 very strong relative advantage, and 9 the ultimate overwhelming relative advantage. Lootsma feels that relative advantage is more naturally concave, and presents a number of cases where a more nearly logarithmic scale would be appropriate, such as planning horizons, loudness of sounds, and brightness of light. Therefore, Lootsma presents a geometric scale where the gradations of decision maker judgment are reflected by the scale as follows:

1/16: *strict preference* for object 2 over base object.
1/4 : *weak preference* for object 2 over the base object.
1 : *indifference*.
4 : *weak preference* for the base object over object 2.
16 : *strict preference* for the base object over object 2.

The ratio of value $r_{jk}$ on the geometric scale is expressed as an exponential function of the difference between the echelons of value on the geometric scale $\delta_{jk}$, as well as a scale parameter $y$. Lootsma considers two alternative scales $y$ to express preferences. For calculating the weight of criteria, $y = \ln \sqrt{2} \approx 0.347$ is used. In REMBRANDT, only one hierarchical level (no matter how many criteria) is used, superior to the level of alternatives. For calculating the weight of alternatives on each criterion, $y = \ln 2 \approx 0.693$ is used. The difference in echelons of value $\delta_{jk}$ is graded as in Table 1, which compares Saaty's ratio scale with the REMBRANDT scale.

The second suggested improvement is the calculation of impact scores. The arithmetic mean is subject to rank reversal of alternatives. The geometric mean is not subject to rank reversal, nor is logarithmic regression. Note that Saaty (1990) argues that rank reversal when new reference points are introduced is a positive feature. Barzilai, Cook and Golany (1987), taking an opposing view, argued that the geometric mean was more appropriate for calculation of relative value (through weights) than the arithmetic mean used by Saaty.

Table 1
AHP scale and corresponding REMBRANDT scale

| Verbal description | Saaty ratio $w_j / w_k$ | REMBRANDT $\delta(jk)$ |
|---|---|---|
| *very strong* preference for object $k$ | 1/9 | −8 |
| *strong* preference for object $k$ | $\frac{1}{7}$ | −6 |
| *definite* preference for object $k$ | 1/5 | −4 |
| *weak* preference for object $k$ | $\frac{1}{3}$ | −2 |
| *indifference* | 1 | 0 |
| *weak* preference for object $j$ | 3 | +2 |
| *definite* preference for object $j$ | 5 | +4 |
| *strong* preference for object $j$ | 7 | +6 |
| *very strong* preference for object $j$ | 9 | +8 |

Lootsma proposes logarithmic regression, minimizing $\Sigma_{j<k}(\ln r_{jk} - \ln v_j + \ln v_k)^2$ where $r_{jk}$ are the ratio comparisons made by the decision maker for base object $j$ and compared object $k$, and the weight for $j$ ($w_j$) is represented by $\ln v_j$. Ratio $r_{jk}$ is the ratio of $w_j/w_k$. The analysis is to calculate these weights. Since $r_{jk} = w_j/w_k$, error is represented by $r_{jk} - w_j/w_k$. The ratio comparisons made by the decision maker are observations, and regression minimizing the squared error yields the set of weights $w_i$ which best fit the decision maker expressed preferences. Solving this is complicated by the fact that the resulting data set is singular. However, a series of normal equations can be solved to yield the desired weights.

To demonstrate, assume a pairwise comparison ratio comparing three factors {A, B and C}, where A is definitely preferred over B, A is strongly preferred over C, while B is weakly preferred over C. This yields the matrix $\delta(jk)$ of preferences, transformed into $r_{jk} = e^{0.347\delta(jk)}$. Weights are desired that minimize the function $\Sigma_{j=1,n}\Sigma_{k=1,n}(\ln r_{jk} - w_j + w_k)^2$. The ratio matrix in REMBRANDT for criteria is transformed through the operator $e^{0.347r(jk)}$ to generate the set of values transformed to the logarithmic scale. Křovác (1987) notes that the geometric means of row elements of such a matrix yields the solution minimizing the sum of squared errors of the form $\Sigma_{j=1,n}\Sigma_{k=1,n}(\ln r_{jk} - w_j + w_k)$. This yields:

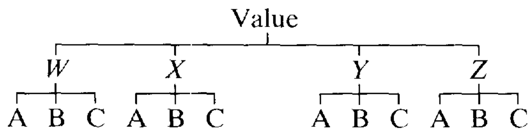
$$\begin{array}{ccc} \delta(jk) & e^{0.347\delta(jk)} & \text{geometric means} \\ \begin{bmatrix} 0 & +4 & +6 \\ -4 & 0 & +2 \\ -6 & -2 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 4 & 8 \\ 0.25 & 1 & 2 \\ 0.125 & 0.5 & 1 \end{bmatrix} & \begin{matrix} 3.175 \\ 0.794 \\ 0.397 \end{matrix} \end{array}$$

This solution is normalized by product. It is a simple matter to normalize by sum, simply dividing each element by the total. REMBRANDT includes a consistency check, in that cases where alternative $j$ is preferred to alternative $k$ on some criteria implies that the resulting score $w_j$ on this criteria should be greater than score $w_k$. If not, the user is informed.

The third improvement proposed by Lootsma is aggregation of scores. REMBRANDT uses one hierarchical level (allowing 25 criteria), with the alternative level (allowing 25 alternatives) subordinate to it. This lowest level is normalized multiplicatively, so that the product of components equals 1 for each of the $k$ factors over which the alternatives are compared. Therefore, each alternative has an estimated relative performance $w_k$ for each of the $k$ factors. The components of the hierarchical level immediately superior to this lowest level are normalized additively, so that they add to 1, yielding weights $O(j)$. The aggregation rule for each alternative $j$ is

$$w_j = \prod_{i=1,k} w_i^{O(i)}.$$

To demonstrate these points, consider an analysis comparing three alternatives, A, B, and C, over four criteria, $W$, $X$, $Y$, and $Z$. The hierarchy may be represented as:




### 2.1. AHP calculations

First we conduct Saaty's AHP analysis. Assume the ratio comparisons of $W$, $X$, $Y$, and $Z$ to be as follows, yielding the eigenvector and inconsistency index (below the limit of 0.09 for four factors, indicating acceptable consistency):

| | $W$ | $X$ | $Y$ | $Z$ | Eigenvector | Inconsistency Index |
|---|---|---|---|---|---|---|
| $W$ | 1 | 1 | 5 | 7 | 0.54922 | 0.0222 |
| $X$ | | 1 | 1 | 3 | 0.32860 | |
| $Y$ | | | 1 | 3 | 0.07259 | |
| $z$ | | | | 1 | 0.04959 | |

This is followed by pairwise comparisons of alternatives A, B, and C on each of the four factors $W$, $X$, $Y$, and $Z$ (the inconsistency indexes are below the limit of 0.06 for three factors, indicating acceptable consistency):

| $W$: | | $A$ | $B$ | $C$ | Eigenvector | Inconsistency Index |
|---|---|---|---|---|---|---|
| | $A$ | 1 | 5 | 7 | 0.73064 | 0.0324 |
| | $B$ | | 1 | 5 | 0.18839 | |
| | $C$ | | | 1 | 0.08096 | |

| $X$: | | $A$ | $B$ | $C$ | | |
|---|---|---|---|---|---|---|
| | $A$ | 1 | $\frac{1}{3}$ | 2 | 0.22965 | 0.0018 |
| | $B$ | | 1 | 5 | 0.64833 | |
| | $C$ | | | 1 | 0.12202 | |

| $Y$: | | $A$ | $B$ | $C$ | | |
|---|---|---|---|---|---|---|
| | $A$ | 1 | 1 | $\frac{1}{5}$ | 0.14884 | 0.0028 |
| | $B$ | | 1 | $\frac{1}{4}$ | 0.16033 | |
| | $C$ | | | 1 | 0.69084 | |

| $Z$: | | $A$ | $B$ | $C$ | | |
|---|---|---|---|---|---|---|
| | $A$ | 1 | 2 | $\frac{1}{2}$ | 0.29696 | 0.0046 |
| | $B$ | | 1 | $\frac{1}{3}$ | 0.16342 | |
| | $C$ | | | 1 | 0.54961 | |

These are then aggregated to obtain weighted scores for each of the alternatives A, B, and C.

$$\text{Value(A)} = 0.54922 * 0.73064 + 0.32860 * 0.22965 + 0.07259 * 0.14884 + 0.04959 * 0.29696 = 0.502,$$

$$\text{Value(B)} = 0.54922 * 0.18839 + 0.32860 * 0.64833 + 0.07259 * 0.16033 + 0.04959 * 0.16342 = 0.336,$$

$$\text{Value(C)} = 0.54922 * 0.08096 + 0.32860 * 0.12202 + 0.07259 * 0.69084 + 0.04959 * 0.53961 = 0.161.$$

These scores can be interpreted as indicating that overall, alternative A has 1.49 times the value of alternative B, and 3.12 times the value of alternative C.

### 2.2. REMBRANDT calculations

The same information can be solved by using geometric means, and the Barzilai, Cook and Golanyi method of aggregation. The following matrices show the $\delta(jk)$ matrices equivalent to Saaty's scale used above, as well as the transformed matrices $e^{0.347\delta(jk)}$, the scores from the row-wise geometric means, and the additively normalized scores:

| | $\delta(jk)$ W | X | Y | Z | | $e^{0.347\delta(jk)}$ W | X | Y | Z | multiplicative | additive |
|---|---|---|---|---|---|---|---|---|---|---|---|
| W | 0 | 0 | 4 | 6 | W | 1 | 1 | 4 | 8 | $\sqrt[4]{1*1*4*8}=2.3784$ | 0.493 |
| X | 0 | 0 | 0 | 2 | X | 1 | 1 | 1 | 2 | $=1.1892$ | 0.246 |
| Y | −4 | 0 | 0 | 2 | Y | 0.25 | 1 | 1 | 2 | $=0.8409$ | 0.174 |
| Z | −6 | −2 | −2 | 0 | Z | 0.125 | 0.5 | 0.5 | 1 | $=0.4204$ | 0.087 |

The additively normal weights are calculated for application as powers over the scores of each alternative. The pairwise comparisons of alternatives A, B, and C on each of the four factors $W$, $X$, $Y$, and $Z$ use an exponential multiplier of ln 2. This yields:

| | | $\delta(jk)$ A | B | C | | $e^{0.693\delta(jk)}$ A | B | C | multiplicative | additive |
|---|---|---|---|---|---|---|---|---|---|---|
| $W$: | A | 0 | −2 | −1 | A | 1 | 16 | 64 | $\sqrt[3]{1*16*64}=10.0794$ | 0.902 |
| | B | −4 | 0 | 4 | B | 0.0625 | 1 | 16 | $=1$ | 0.089 |
| | C | −6 | −4 | 0 | C | 0.015625 | 0.0625 | 1 | $=0.0992$ | 0.009 |
| $X$: | A | 0 | −2 | 1 | A | 1 | 0.25 | 2 | $\sqrt[3]{1*0.25*2}=0.7937$ | 0.155 |
| | B | 2 | 0 | 4 | B | 4 | 1 | 16 | $=4.0$ | 0.783 |
| | C | −1 | −4 | 0 | C | 0.5 | 0.0625 | 1 | $=0.3150$ | 0.062 |
| $Y$: | A | 0 | 0 | −4 | A | 1 | 1 | 0.0625 | $\sqrt[3]{1*1*0.0625}=0.3969$ | 0.067 |
| | B | 0 | 0 | −3 | B | 1 | 1 | 0.125 | $=0.5$ | 0.084 |
| | C | 4 | 3 | 0 | C | 16 | 8 | 1 | $=5.0397$ | 0.849 |
| $Z$: | A | 0 | 1 | −1 | A | 1 | 2 | 0.5 | $\sqrt[3]{1*2*0.5}=1$ | 0.286 |
| | B | −1 | 0 | −2 | B | 0.5 | 1 | 0.25 | $=0.5$ | 0.143 |
| | C | 1 | 2 | 0 | C | 2 | 4 | 1 | $=2$ | 0.571 |

Aggregation is accomplished as:

$$\begin{array}{llllll} A: & 10.0794^{0.493} & *0.7937^{0.246} & *0.3969^{0.174} & *1^{0.087} & =2.513 \quad 0.624, \\ B: & 1^{0.493} & *4^{0.246} & *0.5^{0.174} & *0.5^{0.087} & =1.174 \quad 0.292, \\ C: & 0.0992^{0.493} & *0.3150^{0.246} & *5.0397^{0.174} & *2^{0.087} & =0.339 \quad 0.084. \end{array}$$

Again, the scores reflect relative value. However, REMBRANDT uses a longer scale than AHP, resulting in greater interval distance. Alternative A is 2.14 times as valuable as alternative B using this method, and about 7.43 times as valuable as alternative C.

### 2.3. Group considerations

Both AHP (Aczel and Saaty, 1983) and REMBRANDT (Lootsma, 1992) are capable of supporting group decision making. AHP is often used in group environments (Zahedi, 1986; Shim, 1989). There are in addition some nuances appropriate in group practice that are worth noting. In most applications, the group consensus seems to be used as the basis for one set of pairwise comparisons. In that case, the calculations are straightforward. In Mitchell and Bingham (1986), the application involved a very large group of people. The consensus of managerial pairwise comparisons was used as the basis for top levels of the hierarchy, while the consensus of technical experts was used for evaluating the relative performance of available alternatives on each hierarchical component. A different approach was proposed by Iz (1991, 1992) in presentation of a multiobjective group decision support system. The geometric mean of individual assessments was used as the basis for pairwise comparison inputs, which were then calculated in the traditional eigenvector manner.

In supporting group decisions, support of the process of group interaction is more important than a particular resulting solution. The group should reach a consensus because they agree (at least to a degree) upon the best solution, not because AHP or REMBRANDT or any other technique suggests a particular alternative. The next section presents a case where AHP was applied to a multiobjective group decision problem involving a small group of individuals.

## 3. Group decision – textbook comparison with AHP

Selection of a text is often charged to a committee. In this example, a group of four faculty were given the responsibility of selecting a text to be used by all sections of a core operations management undergraduate course. The group had a variety of objectives. Multiple objective selection techniques have been applied to a number of academic situations to aid in resolution of conflicting views. Lootsma (1980) applied AHP in the selection of a senior professor appointment nomination. Liberatore, Nydick and Sanchez (1992) used AHP to evaluate research papers and select award winners. In each case, these tools seek to reconcile divergent group member opinions. Group members can be presented with all views, and after considering each of these views, a more informed professional judgment can be applied. The aim is to attain a decision arrived at through a consensus. If a consensus is not attained, a mutually acceptable means of making the decision can be applied. The focus of this study is the application of alternative selection techniques which identify differences in order to aid the group process of consensus, or at least agreement on a decision.

### *3.1. Case specifics*

The problem was to adopt an Operations Management textbook for a multiple section undergraduate core business course. Faculty views were elicited relative to sufficiency of textbook topic coverage and the need to provide a means for achieving a group consensus. An objective procedure for evaluating subjective judgments was desired. A six-phase procedure was followed:
1. an initial list of potential texts was developed.
2. the initial long list of texts was screened to identify a short list for deeper evaluation.
3. a specific list of criteria was developed for use in the deeper evaluation, and survey forms were developed,
4. an independent, critical review of each text was conducted by each faculty member,
5. the pairwise comparisons were conducted, and
6. the faculty met to discuss their reviews, and a decision was made.

The first step involved examination of twelve available texts. This was considered too large a set for detailed analysis. Therefore, the second phase reduced this set for deeper analysis. Screening criteria were developed, to include appropriateness for undergraduate students, coverage of important course topics, organization and presentation of material, clarity, available ancillary materials, year of publication, author, etc. Five serious candidates (TeA, TeB, TeC, TeD and TeE) were identified. In the third phase, twelve general topic areas were identified as being essential for core subject material. Each of the five candidate texts provided some degree of coverage over these twelve topics. In phase four, each faculty member was asked to critically review each of the five texts relative to extent and adequacy of topic coverage. In the fifth phase, pairwise comparisons were conducted.

The Appendix presents the twelve criteria agreed upon by the group. These criteria represent the topic areas to be covered. A hierarchy with one level was used, despite efforts to get them to limit the number of branches from one node to seven. The group felt more comfortable considering each of the twelve topics at the same level. This resulted in a tradeoff consisting of more pairwise comparisons, but a more direct evaluation. Given the five alternatives, an additional 120 pairwise comparisons per faculty member were required to evaluate relative performance of each text on each topic. These ratings are appended following the list of twelve criteria.

Because twelve by twelve matrices were used by the decision making group, a FORTRAN code was used for calculations. This code was validated against a code known to be accurate for matrix sizes of seven elements or less. Inconsistency matrices for the twelve by twelve size were not available.

The geometric mean over all group members was used for aggregating group member ratings of relative performance of each alternative text on each topic criterion, as well as for aggregating group member ratings of relative topic importance. This approach follows Iz (1991), and is justified by Barzilai, Cook and Golanyi (1987). Using the geometric mean reduces the impact of extreme individual positions. For instance, two group members may take opposite positions concerning a particular item. One group member may rate the row alternative as 9 times as preferable as the column alternative, while another group member might rate the row alternative as $1/3$ as attractive as the column alternative. The arithmetic mean of these two positions would indicate that the row alternative was 4.67 times as preferable as the column alternative. The geometric mean, on the other hand, would yield a preference rating for the row alternative of 1.73. This score would be a more conservative and stable compromise position for the group. After calculating the pooled geometric means, the inconsistency index used in AHP was found to be very low, indicating high consistency. Using the geometric (or arithmetic) mean will of course reduce the variance in ratings substantially, thereby yielding lower inconsistency ratings. AHP aggregate preference scores are given in Table 2. Two scores are given, one using the conventional arithmetic mean, and the second based upon use of geometric means for calculation of weights (Barzilai, Cook and Golanyi, 1987) in all matrices. Note that using the geometric means yields a different rank order here.

The group agreed to meet and make its decision, having shared the AHP and other more basic ratings of other group members. The group members decided on TeB, while maintaining their individual opinions. There was no difficulty in making the decision, as all group members felt that they had their opportunity to speak, and while the decision between TeB and TeD was very close, no group members strongly disagreed with the final decision.

## 4. REMBRANDT calculations

The REMBRANDT package, written by Leo Rog and provided by Professor Lootsma, was used. The same pairwise comparisons obtained for the AHP analysis (given in the Appendix) were used, converting according to the corresponding elements in Table 1. The REMBRANDT package limits the levels for criteria to one, so the hierarchy developed by the faculty group required no conversion. There was a need to convert some of the pairwise comparison rankings of Professor 3, who used fractional values

Table 2
Aggregate scores by method

*AHP arithmetic mean aggregation:*

| | Prof 1 | Prof 2 | Prof 3 | Prof 4 | GROUP arithmetic | GROUP geometric |
|---|---|---|---|---|---|---|
| TeA | 0.1649 4 | 0.1683 5 | 0.1490 3 | 0.1496 5 | 0.1862 3 | 0.1577 5 |
| TeB | 0.2326 2 | **0.2188** 1 | 0.1264 5 | **0.3074** 1 | **0.2477** 1 | 0.2284 2 |
| TeC | 0.0979 5 | 0.2085 2 | 0.1334 4 | 0.2094 2 | 0.1733 4 | 0.1650 4 |
| TeD | **0.3304** 1 | 0.2036 3 | **0.3955** 1 | 0.1728 3 | 0.2383 2 | **0.2650** 1 |
| TeE | 0.1743 3 | 0.2008 4 | 0.1957 2 | 0.1608 4 | 0.1546 5 | 0.1843 3 |

*REMBRANDT:*

| | Prof 1 | Prof 2 | Prof 3 | Prof 4 | GROUP |
|---|---|---|---|---|---|
| TeA | 0.157 3 | 0.169 5 | 0.117 3 | 0.105 4 | 0.143 5 |
| TeB | 0.238 2 | **0.220** 1 | 0.093 5 | **0.436** 1 | 0.245 2 |
| TeC | 0.083 5 | 0.197 4 | 0.098 4 | 0.213 2 | 0.152 4 |
| TeD | **0.364** 1 | 0.205 3 | **0.542** 1 | 0.148 3 | **0.296** 1 |
| TeE | 0.158 4 | 0.209 2 | 0.151 2 | 0.099 5 | 0.164 3 |

*GROUP scores and rankings summary:*

| | AHP arithmetic | AHP geometric | REMBRANDT |
|---|---|---|---|
| TeA | 0.186 3 | 0.158 5 | 0.143 5 |
| TeB | **0.247** 1 | 0.228 2 | 0.245 2 |
| TeC | 0.173 4 | 0.165 4 | 0.152 4 |
| TeD | 0.238 2 | **0.265** 1 | **0.296** 1 |
| TeE | 0.155 5 | 0.184 3 | 0.164 3 |

greater than one (for instance, in comparing textbooks on topics of facility layout planning, forecasting, aggregate production planning, independent demand inventory, and project management). The FORTRAN package used for AHP calculations was capable of taking such fractional values. But REMBRANDT provides the user with a given scale to be selected from, requiring integer values. Fractions were rounded to the nearest integer, and where there was a tie (for either the rating or its reciprocal), the fraction was rounded up.

Comparative results shown in Table 2 indicate that group results obtained by REMBRANDT were very similar to those obtained using the geometric mean in AHP. Both were quite different from those yielded by AHP using the conventional arithmetic mean aggregation rule. This would imply that the scale used seems to have less impact than the aggregation rule. As far as any impact on group decision making members, there would be no effect, because the input would be precisely the same. The only differences would be in theoretical characteristics of the different approaches. The geometric mean aggregation rule avoids rank reversal, which has varying degrees of importance to different researchers (Dyer, 1990a; Saaty, 1990; Harker and Vargas, 1990; Dyer, 1990b). The geometric mean approaches, as used in REMBRANDT, avoid rank reversal.

AHP has an inconsistency index which reports inconsistencies above those found in 90 percent of random matrices. REMBRANDT has no index per se, but does report cases where direct rating ranked two elements one way, and the resulting weights yielded reversed rank order. In the case used here, there were five such inconsistencies noted by REMBRANDT. For Professor 1, LoP was rated as twice as important as F. REMBRANDT reported the weight for LoP was 0.078, and the weight for F was 0.095, indicating some inconsistency. An inconsistency was also reported by REMBRANDT for Professor 2's ratings of PM (rated twice as important as JIT), while the final score for JIT was 0.096 and that for PM was 0.076. Professor 3 rated TeB as 4/3 as attractive as TeA relative to IDI. But the scores for that matrix yielded 0.200 for TeA and 0.100 for TeB. The inconsistency index for the AHP index for that matrix was 0.0550, below the conventional limit. Professor 4 rated TeB as twice as useful as TeC relative to TQM, and TeB as twice as useful as TeD on the same topic. But the score obtained for TQM by TeB was 0.192, while TeC obtained a score of 0.254 and TeD a score of 0.254 as well. The AHP inconsistency index for this matrix was 0.1104, below the conventional limit of 0.112. The point is that while REMBRANDT does not have a developed inconsistency index, it is programmed to identify potential inconsistencies which appear to be as precise as the conventional AHP inconsistency index. Furthermore, REMBRANDT provides additional help in identifying the source of the inconsistency.

## 5. Analysis of differences

The group had distinctly different opinions. Professor 3 had a strong preference for TeD. Professor 4 favored TeB, followed by TeC. Professor 1 distinctly favored TeD followed by TeB. Professor 2 had much closer views on all five texts. These differences were apparently based upon fundamental opinions, because none of the participants indicated any desire to change their personal ratings when presented evidence about these differences. One benefit offered through use of these methods to support the group process results from identifying sources of group differences. For instance, it is apparent that the group members have different views about the relative importance of forecasting (F) and aggregate planning (AgP). Professor 3 rates AgP as important as any other topic, while F is rated the lowest of any topic. On the other hand, professor 4 rates F as important as any topic, while AgP is tied for the least important. Professors 1 and 2 both rate forecasting relatively high, and aggregate planning relatively low. Differences of opinion such as this are features of group interaction which need to be considered when selecting a method for support. The geometric mean (used by REMBRANDT) is influenced more by strongly held positions than is the arithmetic mean (used by AHP). Therefore, the two methods yielded different suggested solutions. Eigenvector rankings of the individual group members indicated Professors 1 and 3 preferring TeD, while Professors 2 and 4 preferred TeB. Developing a group rating based upon the arithmetic mean yields TeB as the highest rated alternative, closely followed by TeD. On the other hand, using the geometric mean to develop a group rating (as in Iz, 1991) results in TeD receiving a notably higher rating than TeB.

The REMBRANDT system would carry the analysis a step further by developing geometric means on the basis of each individual rating. This results in the same individual rating selections. The group selection would be TeD, by a slightly higher margin than was obtained by using the geometric means.

The implications of these differences are not major. Both approaches provide a basis for supporting group decision making, with some attractive features. The primary benefit is that differences of opinion can easily be identified. Further, if normal group processes that could lead to consensus are followed, these differences do not necessarily have to be reconciled. The methods both have means of aggregating these different opinions.

## 6. Conclusions

This study compared the use of REMBRANDT with AHP in a group selection problem. The input for both systems is identical, with the exception that the verbal scale given in the appendix has different numerical values assigned. Both systems are well suited to group decisions. There are a number of questions about aggregating group preference statements when using AHP, but a number of approaches have been published, with some critical mass appearing to have developed around the use of geometric means to aggregate individual ratings (Iz, 1991).

The differences between the systems relate to their procedural operations. First, REMBRANDT replaces the ratio scale suggested by Saaty with two longer scales. For this study, this difference led to more divergent individual ratings by Professors 3 and 4, who held extreme positions. There was practically no difference in scores for Professors 1 and 2. The group scores also were more divergent for REMBRANDT.

The second procedural difference was in calculation of impact scores. Use of the geometric mean in the AHP analysis yielded a different recommended solution than use of the arithmetic mean. Ranks of the first two alternatives were in fact reversed. As might be expected in cases when ranks are reversed, the reversed alternatives were very close in scores with both methods. The REMBRANDT system yielded the same rank order as AHP with geometric calculation of impact scores. There was a slightly more divergent scoring of the alternatives, which can be explained by the use of longer scales.

The third procedural difference is in aggregation of alternative scores over criteria. While this also may have had some impact in the greater divergence of scores between REMBRANDT and AHP, the relative impact seems to be minor. The aggregation rule used by REMBRANDT is appropriate because it makes theoretical sense (fitting the exponential form of the input).

We consider both AHP and REMBRANDT to be effective tools for aiding group selection decisions faced with multiple criteria. AHP is more flexible, in that multilevel hierarchies are possible. But that is a very minor advantage, if any. In this case, the group actually insisted on one level for twelve hierarchical elements when using AHP. REMBRANDT forces such a hierarchy as well. REMBRANDT is designed with group decisions in mind, with program capacity for up to 250 group members. (A group of one is also allowable.)

In the multiobjective decision problem reviewed here, consensus was not fully reached, although the group was happy with their decision. We do not think that any system can guarantee group convergence on a choice. But systems such as AHP and REMBRANDT provide a framework where individuals can express their opinions on detailed elements of the decision. These individual opinions can easily be shared to identify differences of opinion. The group in question felt more comfortable making the decision the traditional way, through voting, and settling on a decision when that vote was tied. In the spirit of decision support systems, this was much preferred over letting the numbers make the decision. And we feel that the results of this study confirm this approach, as the two methods examined yielded the two different alternatives supported by group members. The analysis was considered valuable by group members because of its structure in analyzing the decision.

## Appendix

| Core Course Topics: | |
|---|---|
| Operations strategy | OS |
| Total quality management and statistical quality control | TQM |
| Facility capacity planning | CP |
| Facility location planning | LoP |
| Facility layout planning | LaP |
| Demand forecasting | F |
| Aggregate production planning | AgP |

| | |
|---|---|
| Master production scheduling | MPS |
| Material requirements planning | MRP |
| Independent demand inventory planning and control | IDI |
| Just-in-time materials and purchasing management | JIT |
| Project management | PM |

## INDIVIDUAL PAIRWISE COMPARISONS:

| *Prof 1* | OS | TQM | CP | LoP | LaP | F | AgP | MPS | MRP | IDI | JIT | PM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OS | 1 | 1 | 8 | 3 | 9 | 3 | 6 | 3 | 4 | 7 | 3 | 3 |
| TQM | | 1 | 7 | 3 | 8 | 2 | 5 | 6 | 3 | 5 | 2 | 2 |
| CP | | | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{3}$ | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ |
| LoP | | | | 1 | 4 | 2 | 1 | 2 | 2 | 3 | 1 | 1 |
| LaP | | | | | 1 | $\frac{1}{5}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{6}$ |
| F | | | | | | 1 | 5 | 3 | 2 | 4 | 1 | 1 |
| AgP | | | | | | | 1 | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ |
| MPS | | | | | | | | 1 | $\frac{1}{3}$ | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ |
| MRP | | | | | | | | | 1 | 3 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| IDI | | | | | | | | | | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ |
| JIT | | | | | | | | | | | 1 | 1 |
| PM | | | | | | | | | | | | 1 |

| *Prof 2* | OS | TQM | CP | LoP | LaP | F | AgP | MPS | MRP | IDI | JIT | PM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OS | 1 | 2 | 3 | 4 | 3 | 2 | 3 | 3 | 3 | 3 | 2 | 3 |
| TQM | | 1 | 2 | 3 | 2 | 1 | 2 | 2 | 2 | 2 | 1 | 2 |
| CP | | | 1 | 2 | 1 | $\frac{1}{2}$ | 1 | 1 | 1 | 1 | $\frac{1}{2}$ | 1 |
| LoP | | | | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{2}$ |
| LaP | | | | | 1 | $\frac{1}{2}$ | 1 | 1 | 1 | 1 | $\frac{1}{2}$ | 1 |
| F | | | | | | 1 | 2 | 2 | 2 | 2 | 1 | 2 |
| AgP | | | | | | | 1 | 1 | 1 | 1 | $\frac{1}{2}$ | 1 |
| MPS | | | | | | | | 1 | 1 | 1 | $\frac{1}{2}$ | 1 |
| MRP | | | | | | | | | 1 | 1 | $\frac{1}{2}$ | 1 |
| IDI | | | | | | | | | | 1 | $\frac{1}{2}$ | 1 |
| JIT | | | | | | | | | | | 1 | $\frac{1}{2}$ |
| PM | | | | | | | | | | | | 1 |

| *Prof 3* | OS | TQM | CP | LoP | LaP | F | AgP | MPS | MRP | IDI | JIT | PM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OS | 1 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 2 |
| TQM | | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 2 |
| CP | | | 1 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 2 |
| LoP | | | | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 2 |
| LaP | | | | | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 2 |
| F | | | | | | 1 | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |
| AgP | | | | | | | 1 | 1 | 1 | 1 | 1 | 2 |
| MPS | | | | | | | | 1 | 1 | 1 | 1 | 2 |
| MRP | | | | | | | | | 1 | 1 | 1 | 2 |
| IDI | | | | | | | | | | 1 | 1 | 2 |
| JIT | | | | | | | | | | | 1 | 2 |
| PM | | | | | | | | | | | | 1 |

| *Prof 4* | OS | TQM | CP | LoP | LaP | F | AgP | MPS | MRP | IDI | JIT | PM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OS | 1 | 1/4 | 4 | 4 | 4 | 1 | 4 | 4 | 4 | 1 | 1 | 1 |
| TQM | | 1 | 7 | 7 | 7 | 4 | 7 | 7 | 7 | 4 | 4 | 4 |
| CP | | | 1 | 1 | 1 | $\frac{1}{4}$ | 1 | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| LoP | | | | 1 | 1 | $\frac{1}{4}$ | 1 | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| LaP | | | | | 1 | $\frac{1}{4}$ | 1 | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| F | | | | | | 1 | 4 | 4 | 4 | 1 | 1 | 1 |
| AgP | | | | | | | 1 | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| MPS | | | | | | | | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| MRP | | | | | | | | | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| IDI | | | | | | | | | | 1 | 1 | 1 |
| JIT | | | | | | | | | | | 1 | 1 |
| PM | | | | | | | | | | | | 1 |

*Operations Strategy:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 4 | $\frac{1}{2}$ | 1 |
| TeB | | 1 | 2 | $\frac{1}{2}$ | 1 |
| TeC | | | 1 | $\frac{1}{4}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 4 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 1 |
| TeB | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | 2 |
| TeC | | | 1 | 1 | 2 |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | 2 |
| TeB | | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | 2 |
| TeC | | | 1 | $\frac{1}{2}$ | 4 |
| TeD | | | | 1 | 8 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{7}$ | $\frac{1}{7}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |
| TeB | | 1 | 1 | 3 | 4 |
| TeC | | | 1 | 4 | 4 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

*Total Quality Management:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 2 | $\frac{1}{4}$ | 1 |
| TeB | | 1 | 2 | $\frac{1}{2}$ | 1 |
| TeC | | | 1 | $\frac{1}{4}$ | 1 |
| TeD | | | | 1 | 4 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 2 | 2 | 2 |
| TeB | | 1 | 2 | 2 | 2 |
| TeC | | | 1 | 1/2 | 1 |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | 2 |
| TeB | | 1 | 2 | $\frac{1}{2}$ | 4 |
| TeC | | | 1 | $\frac{1}{4}$ | 2 |
| TeD | | | | 1 | 8 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | 1 | 7 |
| TeB | | 1 | 2 | 2 | 2 |
| TeC | | | 1 | 1 | 7 |
| TeD | | | | 1 | 7 |
| TeE | | | | | 1 |

*Capacity Planning:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | 1 | 1 |
| TeB | | 1 | 1 | 1 | 1 |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 2 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeB | | 1 | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 2 | 1 | $\frac{1}{2}$ | 1 |
| TeB | | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |
| TeC | | | 1 | $\frac{1}{2}$ | 1 |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | 1 | 1 |
| TeB | | 1 | 1 | 1 | 1 |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

*Location Planning:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 1 |
| TeB | | 1 | 3 | 1 | 2 |
| TeC | | | 1 | $\frac{1}{2}$ | 1 |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{3}$ | 1 | 1 | $\frac{1}{3}$ |
| TeB | | 1 | 3 | 3 | 1 |
| TeC | | | 1 | 1 | $\frac{1}{3}$ |
| TeD | | | | 1 | $\frac{1}{3}$ |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | $\frac{1}{4}$ | 1 |
| TeB | | 1 | 1 | $\frac{1}{4}$ | 1 |
| TeC | | | 1 | $\frac{1}{4}$ | 1 |
| TeD | | | | 1 | 4 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 3 | 3 | 3 |
| TeB | | 1 | 3 | 3 | 3 |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

*Layout Planning:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | 1 | 1 |
| TeB | | 1 | 1 | 1 | 1 |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 2 | 1 | 1 |
| TeB | | 1 | 3 | 2 | 2 |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{4}$ | $\frac{1}{3}$ |
| TeB | | 1 | 2 | $\frac{1}{2}$ | $\frac{2}{3}$ |
| TeC | | | 1 | $\frac{1}{4}$ | $\frac{1}{3}$ |
| TeD | | | | 1 | $\frac{4}{3}$ |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{4}$ | $\frac{1}{3}$ | 1 | 1 |
| TeB | | 1 | 1 | 3 | 3 |
| TeC | | | 1 | 4 | 4 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

*Forecasting:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 3 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeB | | 1 | 3 | 1 | 2 |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeB | | 1 | 1 | 2 | 2 |
| TeC | | | 1 | 2 | 2 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | $\frac{3}{2}$ | $\frac{3}{5}$ | $\frac{3}{2}$ |
| TeB | | 1 | $\frac{3}{2}$ | $\frac{3}{5}$ | $\frac{3}{2}$ |
| TeC | | | 1 | $\frac{2}{5}$ | 1 |
| TeD | | | | 1 | $\frac{5}{2}$ |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | 1 | 1 |
| TeB | | 1 | 1 | 1 | 1 |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

*Aggregate Production Planning:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 2 | $\frac{1}{2}$ | 1 |
| TeB | | 1 | 2 | 1 | 2 |
| TeC | | | 1 | $\frac{1}{3}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 2 | 1 | 1 | 1 |
| TeB | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |
| TeB | | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{2}{3}$ |
| TeD | | | | 1 | $\frac{4}{3}$ |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | $\frac{1}{3}$ | 5 | 1 |
| TeB | | 1 | $\frac{1}{3}$ | 7 | 1 |
| TeC | | | 1 | 8 | 4 |
| TeD | | | | 1 | $\frac{1}{7}$ |
| TeE | | | | | 1 |

*MPS:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 2 | $\frac{1}{2}$ | 1 |
| TeB | | 1 | 2 | 1 | 2 |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | $\frac{1}{2}$ | 1 | 2 |
| TeB | | 1 | $\frac{1}{2}$ | 1 | 2 |
| TeC | | | 1 | 2 | 3 |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | $\frac{1}{3}$ | 1 |
| TeB | | 1 | 1 | $\frac{1}{3}$ | 1 |
| TeC | | | 1 | $\frac{1}{3}$ | 1 |
| TeD | | | | 1 | 3 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 1 |
| TeB | | 1 | 1 | 1 | 4 |
| TeC | | | 1 | 1 | 4 |
| TeD | | | | 1 | 4 |
| TeE | | | | | 1 |

*Materials Requirements Planning:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{3}$ | $\frac{1}{2}$ |
| TeB | | 1 | 3 | 1 | 1 |
| TeC | | | 1 | $\frac{1}{3}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | 1 | 2 |
| TeB | | 1 | 1 | 1 | 2 |
| TeC | | | 1 | 1 | 2 |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 4 | 4 | 1 | 1 |
| TeB | | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ |
| TeC | | | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 1 | $\frac{1}{4}$ | 1 |
| TeB | | 1 | 1 | $\frac{1}{4}$ | 1 |
| TeC | | | 1 | $\frac{1}{4}$ | 1 |
| TeD | | | | 1 | 4 |
| TeE | | | | | 1 |

*Independent Demand Inventory:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 2 | $\frac{1}{3}$ | 1 |
| TeB | | 1 | 3 | 1 | 2 |
| TeC | | | 1 | $\frac{1}{3}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 2 | 1 | 2 | $\frac{1}{2}$ |
| TeB | | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{3}$ |
| TeC | | | 1 | 2 | $\frac{1}{2}$ |
| TeD | | | | 1 | $\frac{1}{3}$ |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{3}{4}$ | $\frac{3}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| TeB | | 1 | 1 | $\frac{1}{4}$ | $\frac{1}{4}$ |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{7}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{6}$ |
| TeB | | 1 | 3 | 3 | 1 |
| TeC | | | 1 | 1 | $\frac{1}{4}$ |
| TeD | | | | 1 | $\frac{1}{4}$ |
| TeE | | | | | 1 |

*Just In Time:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 3 | 1 | $\frac{1}{2}$ |
| TeB | | 1 | 2 | 1 | 1 |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 1 | 3 | 2 | $\frac{1}{2}$ |
| TeB | | 1 | 3 | 2 | $\frac{1}{2}$ |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ |
| TeD | | | | 1 | $\frac{1}{3}$ |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | 2 | 2 | $\frac{1}{4}$ | $\frac{1}{2}$ |
| TeB | | 1 | 1 | $\frac{1}{8}$ | $\frac{1}{4}$ |
| TeC | | | 1 | $\frac{1}{8}$ | $\frac{1}{4}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{7}$ | 1 | 1 | $\frac{1}{6}$ |
| TeB | | 1 | 7 | 7 | 1 |
| TeC | | | 1 | 1 | $\frac{1}{7}$ |
| TeD | | | | 1 | $\frac{1}{7}$ |
| TeE | | | | | 1 |

*Project Management:*

| *Prof 1* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{3}$ | 3 | $\frac{1}{2}$ | 1 |
| TeB | | 1 | 3 | 1 | 1 |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 2 |
| TeE | | | | | 1 |

| *Prof 2* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeB | | 1 | 2 | 1 | 1 |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 3* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
| TeB | | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeC | | | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

| *Prof 4* | TeA | TeB | TeC | TeD | TeE |
|---|---|---|---|---|---|
| TeA | 1 | $\frac{1}{5}$ | 1 | 1 | 1 |
| TeB | | 1 | 4 | 4 | 4 |
| TeC | | | 1 | 1 | 1 |
| TeD | | | | 1 | 1 |
| TeE | | | | | 1 |

**GEOMETRIC MEANS OF INDIVIDUAL RANKINGS – EIGENVECTOR**

| | OS | TQM | CP | LoP | LaP | F | AgP | MPS | MRP | IDI | JIT | PM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OS | 1 | 0.841 | 3.130 | 3.364 | 3.224 | 2.060 | 2.913 | 2.450 | 2.632 | 2.141 | 1.565 | 2.060 |
| TQM | | 1 | 3.146 | 2.817 | 3.253 | 2.213 | 2.893 | 3.027 | 2.546 | 2.515 | 1.682 | 2.378 |
| CP | | | 1 | 1 | 1 | 0.595 | 0.841 | 1 | 0.841 | 0.707 | 0.452 | 0.595 |
| LoP | | | | 1 | 1.189 | 0.841 | 0.841 | 1 | 1 | 0.783 | 0.537 | 0.707 |
| LaP | | | | | 1 | 0.523 | 0.841 | 0.707 | 0.707 | 0.473 | 0.398 | 0.537 |
| F | | | | | | 1 | 1.911 | 1.682 | 1.520 | 1.278 | 0.760 | 1.075 |
| AgP | | | | | | | 1 | 1 | 0.841 | 0.707 | 0.420 | 0.595 |
| MPS | | | | | | | | 1 | 0.760 | 0.707 | 0.420 | 0.595 |
| MRP | | | | | | | | | 1 | 0.931 | 0.500 | 0.707 |
| IDI | | | | | | | | | | 1 | 0.595 | 0.841 |
| JIT | | | | | | | | | | | 1 | 1 |
| PM | | | | | | | | | | | | 1 |
| eigen | 0.1588 | 0.1699 | 0.0503 | 0.0559 | 0.0446 | 0.0852 | 0.0533 | 0.0532 | 0.0613 | 0.0722 | 0.1107 | 0.0847 |

**REMBRANDT RESULTS**

*Prof 1:*

| | criterion weight | TeA | teb | TeC | TeD | TeE |
|---|---|---|---|---|---|---|
| Operations Strategy | 0.208 | 0.205 | 0.155 | 0.051 | 0.471 | 0.118 |
| Total Quality Management | 0.175 | 0.095 | 0.165 | 0.072 | 0.574 | 0.095 |
| Capacity Planning | 0.036 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 |
| Location Planning | 0.078 | 0.140 | 0.321 | 0.121 | 0.279 | 0.140 |
| Layout Planning | 0.020 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 |
| Forecasting | 0.095 | 0.156 | 0.313 | 0.078 | 0.272 | 0.180 |
| Aggregate Production Planning | 0.040 | 0.158 | 0.276 | 0.091 | 0.317 | 0.158 |
| Master Production Scheduling | 0.044 | 0.163 | 0.283 | 0.107 | 0.283 | 0.163 |
| Materials Requirements Planning | 0.067 | 0.100 | 0.264 | 0.087 | 0.349 | 0.200 |
| Independent Demand Inventory | 0.039 | 0.131 | 0.300 | 0.075 | 0.345 | 0.150 |
| Just In Time | 0.095 | 0.185 | 0.243 | 0.080 | 0.212 | 0.280 |
| Project Management | 0.104 | 0.156 | 0.313 | 0.078 | 0.272 | 0.180 |
| aggregate score | | 0.157 | 0.238 | 0.083 | 0.364 | 0.158 |

*Prof 2:*

| | criterion weight | TeA | teb | TeC | TeD | TeE |
|---|---|---|---|---|---|---|
| Operations Strategy | 0.144 | 0.089 | 0.177 | 0.309 | 0.309 | 0.117 |
| Total Quality Management | 0.102 | 0.283 | 0.283 | 0.123 | 0.187 | 0.123 |
| Capacity Planning | 0.072 | 0.133 | 0.067 | 0.267 | 0.267 | 0.267 |
| Location Planning | 0.051 | 0.091 | 0.364 | 0.091 | 0.091 | 0.364 |
| Layout Planning | 0.072 | 0.182 | 0.364 | 0.091 | 0.182 | 0.182 |
| Forecasting | 0.102 | 0.077 | 0.308 | 0.308 | 0.154 | 0.154 |
| Aggregate Production Planning | 0.072 | 0.222 | 0.111 | 0.222 | 0.222 | 0.222 |
| Master Production Scheduling | 0.072 | 0.182 | 0.182 | 0.364 | 0.182 | 0.091 |
| Materials Requirements Planning | 0.072 | 0.222 | 0.222 | 0.222 | 0.222 | 0.111 |
| Independent Demand Inventory | 0.072 | 0.200 | 0.100 | 0.200 | 0.100 | 0.400 |
| Just In Time | 0.096 | 0.211 | 0.211 | 0.053 | 0.105 | 0.421 |
| Project Management | 0.076 | 0.125 | 0.250 | 0.125 | 0.250 | 0.250 |
| aggregate score | | 0.169 | 0.220 | 0.197 | 0.205 | 0.209 |

*Prof 3:*

| | criterion weight | TeA | teb | TeC | TeD | TeE |
|---|---|---|---|---|---|---|
| Operations Strategy | 0.089 | 0.065 | 0.065 | 0.170 | 0.682 | 0.019 |
| Total Quality Management | 0.089 | 0.065 | 0.170 | 0.065 | 0.682 | 0.019 |
| Capacity Planning | 0.089 | 0.174 | 0.076 | 0.174 | 0.401 | 0.174 |
| Location Planning | 0.089 | 0.083 | 0.083 | 0.083 | 0.667 | 0.083 |
| Layout Planning | 0.089 | 0.067 | 0.154 | 0.067 | 0.405 | 0.307 |
| Forecasting | 0.045 | 0.208 | 0.208 | 0.119 | 0.362 | 0.104 |
| Aggregate Production Planning | 0.089 | 0.067 | 0.067 | 0.154 | 0.405 | 0.307 |
| Master Production Scheduling | 0.089 | 0.125 | 0.125 | 0.125 | 0.500 | 0.125 |
| Materials Requirements Planning | 0.089 | 0.308 | 0.038 | 0.038 | 0.308 | 0.308 |
| Independent Demand Inventory | 0.089 | 0.128 | 0.084 | 0.111 | 0.338 | 0.338 |
| Just In Time | 0.089 | 0.049 | 0.014 | 0.014 | 0.776 | 0.147 |
| Project Management | 0.063 | 0.142 | 0.164 | 0.124 | 0.285 | 0.285 |
| aggregate score | | 0.117 | 0.093 | 0.098 | 0.542 | 0.151 |

*Prof 4:*

| | criterion weight | TeA | teb | TeC | TeD | TeE |
|---|---|---|---|---|---|---|
| Operations Strategy | 0.101 | 0.008 | 0.407 | 0.467 | 0.067 | 0.051 |
| Total Quality Management | 0.284 | 0.291 | 0.192 | 0.254 | 0.254 | 0.009 |
| Capacity Planning | 0.036 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 |
| Location Planning | 0.036 | 0.364 | 0.364 | 0.091 | 0.091 | 0.091 |
| Layout Planning | 0.036 | 0.070 | 0.368 | 0.423 | 0.070 | 0.070 |
| Forecasting | 0.101 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 |
| Aggregate Production Planning | 0.036 | 0.114 | 0.150 | 0.601 | 0.004 | 0.131 |
| Master Production Scheduling | 0.036 | 0.054 | 0.330 | 0.287 | 0.287 | 0.041 |
| Materials Requirements Planning | 0.036 | 0.083 | 0.083 | 0.083 | 0.667 | 0.083 |
| Independent Demand Inventory | 0.101 | 0.009 | 0.392 | 0.074 | 0.074 | 0.450 |
| Just In Time | 0.101 | 0.009 | 0.521 | 0.008 | 0.008 | 0.453 |
| Project Management | 0.101 | 0.067 | 0.704 | 0.077 | 0.077 | 0.077 |
| aggregate score | | 0.105 | 0.436 | 0.213 | 0.148 | 0.099 |

| *aggregate for group:* | criterion weight | TeA | teb | TeC | TeD | TeE |
|---|---|---|---|---|---|---|
| Operations Strategy | 0.136 | 0.074 | 0.218 | 0.250 | 0.379 | 0.080 |
| Total Quality Management | 0.155 | 0.173 | 0.229 | 0.127 | 0.427 | 0.043 |
| Capacity Planning | 0.057 | 0.181 | 0.124 | 0.215 | 0.265 | 0.215 |
| Location Planning | 0.063 | 0.171 | 0.298 | 0.117 | 0.242 | 0.171 |
| Layout Planning | 0.049 | 0.132 | 0.293 | 0.174 | 0.207 | 0.193 |
| Forecasting | 0.087 | 0.158 | 0.266 | 0.164 | 0.248 | 0.164 |
| Aggregate Production Planning | 0.059 | 0.168 | 0.174 | 0.273 | 0.132 | 0.254 |
| Master Production Scheduling | 0.060 | 0.130 | 0.235 | 0.212 | 0.321 | 0.102 |
| Materials Requirements Planning | 0.067 | 0.178 | 0.135 | 0.102 | 0.408 | 0.178 |
| Independent Demand Inventory | 0.075 | 0.089 | 0.212 | 0.126 | 0.205 | 0.369 |
| Just In Time | 0.101 | 0.102 | 0.218 | 0.041 | 0.171 | 0.468 |
| Project Management | 0.090 | 0.130 | 0.344 | 0.110 | 0.219 | 0.197 |
| aggregate score | | 0.143 | 0.245 | 0.152 | 0.296 | 0.164 |

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