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Comparative Analysis of Multiattribute Techniques Based on Cardinal and Ordinal Inputs

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Abstract—This paper compares multiattribute decision analysis under conditions of partial information and ordinal input Difficult decisions based on partial information usually are dealt with through obtaining more precise input information The purpose of this paper is to present a technique for systematically exploring the entire region within weight bounds established by ordinal input data The center of mass of the product of weights and utilities is used Some consideration of sensitivity analysis for this problem is presented © 2001 Elsevier Science Ltd All rights reserved

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1. INTRODUCTION

Selection models are an important field within attribute analysis This area includes multiattribute utility analysis (MAUT [1,2]), the simple multiattribute rating technique (SMART [3,4]), analytic hierarchy process [5], and other applications These methods use cardinal weight input information

In multiattribute decision making, the derivation of weights is often a central step in eliciting decision-maker preferences [6] The difficulties in assessing preference weights have been widely noted [7–9] Existing methods try to infer human preferences based on exact statements and evaluations—regardless of whether the humans involved have a clear understanding of the questions that they are asked Weber [10] argues that decision-maker preferences are rarely structured enough to allow the successful application of most decision analysis methods Kirkwood and Sarin [11] presented an approach to use partial weight and utility information as a means to weed out clearly inferior alternatives before investing thorough analysis on the more attractive

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alternatives Podinovski [12] gave four reasons why precise evaluation of tradeoffs may often be difficult or even impossible

(1) information about the relative importance of criteria may be insufficient,
(2) tradeoffs may be different for different levels of criteria values,
(3) the problem may be analyzed from different perspectives, or
(4) different experts may specify different tradeoffs

Kirkwood and Corner [13] suggested that the use of approximate weights would simplify decision analysis, since detailed elicitation of weights can be both time consuming and inconsistent Sensitivity analysis of weights is often insightful [14,15] Hauser and Tadikamalla [16] argued that the analysis of inconsistency may reveal useful information regarding the overall importance of some uncertain judgements

The centroid approach [17–22] uses ordinal input information about relative weights rather than cardinal input as used in MAUT and SMART Ordinal input information is expected to be more robust While less precise numerically, the ability of humans to state ordinal ranking is considered more reliable than precise ratio statements of input [23]

The linear utility function model used in SMART and centroid approaches is

$$\text{Maximize} \sum_{j=1}^{k} w_j u_{ij}, \qquad \forall i = 1 \text{ to } n, \tag{1}$$

where w_j is the scaling value (weight) assigned to the jth of k criteria, and u_{ij} is the utility for alternative i on criterion j The selection decision is to identify which of the n alternatives have the maximum value function This value function also can be used to rank order the n alternatives

SMART and AHP use the same overall model, but differ in how estimates of the model components w_j and u_{ij} are determined SMART allows the decision maker to estimate both w_j and u_{ij} directly on a 0–1 scale Edwards and Barron [19] presented swing weighting in the SMART approach, using the same model described above, but based on a controlled means of estimating the criteria weights w_j AHP uses eigenvalues of ratio pairwise comparisons for both w_j and u_{ij}, yielding estimates ranging between 0 and 1

Solymosi and Dombi [22] presented a technique using interactive elicitation of preference weights among pairs of criteria The core of the method is that preference information among criteria provides knowledge about the bounds of specific weight values They used the centroid of this bounded area as a likely estimate of true weights The centroid method (SMARTER, in [19]) uses the same overall model as SMART, only using ordinal input information Maximum error is minimized While continuous weight estimation methods, such as multiattribute utility theory models or analytic hierarchy models, would be expected to be more accurate estimators if preference input were accurate, the centroid approach is based on sounder input, and is less subject to the errors introduced by inaccurate weight assessment Flores *et al* [24] found that the centroid approach was useful when there were four or more criteria being considered, when criteria were close in relative importance, and when time available for analysis was short

There is an important issue that has not been examined and implemented until now Although the centroid approach considers bounds on specific weight values, it uses the centroid point of weights only Estimation of this centroid point is only one possible way to use information about the configuration of the bounded area In this paper, we first comparatively demonstrate SMART, then centroid, and then a new method using ordinal input information about weights as well as utility measures on each criterion

The purpose of this paper is to extend the centroid approach to explore the entire region within weight bounds based on ordinal input, and to examine sensitivity analysis in centroid models considering utilities This may allow deeper evaluation of existing alternatives The

paper also examines the improvement of values in the current conditional utilities of existing alternatives needed to raise alternative performance to the level where it is clearly preferable to the other alternatives

2. SMART

The simple multiattribute rating technique uses a linear additive model to estimate the value of each alternative as discussed above The method begins with identifying the decision and the responsible decision-maker (Step 1), the issues important in the decision (criteria, Step 2), and the alternatives available (Step 3) Each criterion's measurement scale is established in Step 4, along with measures as given in the table above Step 5 is to eliminate dominated alternatives (one alternative dominates another if its performance is at least as good as the dominated alternative on all criteria, and better on at least one criterion) Step 6 is to develop single-attribute utilities, reflecting how well each alternative does on each criterion In Step 7, swing weighting is applied to determine weights for the linear additive model This operation begins with rank-ordering criteria, considering their measurement scales The decision-maker is asked to compare two criteria, beginning with identifying which criterion would be most attractive to improve from the worst attainment considered to the best attainment considered This provides a basis for rank-ordering criteria (after considering scale) Step 8 would be to obtain estimates of relative weights by comparing the most important criterion with each of the others, by asking the decision-maker to assess how important the other criteria would be should the most important criteria be worth 100 Weights are obtained by normalizing (sum the assessed values, and divide each value by the sum) The last step (Step 9) of the swing-weighting approach is to obtain values for each alternative using the formula given above (sum of products of each weight times utility values for each alternative)

We use an example decision of siting a new facility There are seven alternative locations available, with four criteria important to the decision Cost (in millions of dollars) is to be minimized Growth potential (in thousands of potential customers) and skilled labor available are to be maximized Transportation availability is a subjectively measured concept The matrix of alternative attainments is presented in Table 1

Table 1

	Cost ($ Million)	Customer (Thousands)	Skilled Labor (Workers)	Transportation
New York	13 6	500	3000	great
Los Angeles	12 8	600	3600	great
Phoenix	11 6	480	2800	good
Houston	11 4	450	2900	good
Denver	11 2	370	2600	good
Dallas	10 9	350	2400	fair
Nashville	10 6	280	1200	poor

As applied to the decision problem given above, the decision is to select a site for a decision maker The objective hierarchy is simply the four criteria The seven alternatives given in the table above identify the alternatives, as well as the dimensions by attributes matrix The Los Angeles site dominates the New York site (LA is better on cost, growth potential, and skill availability, while the two alternatives have equal ratings on transportation availability), so the New York site could be eliminated However, the decision-maker might be interested in seeing the relative performance of New York, so we will keep the New York site for analysis

Step 6 is to develop single-dimension utilities For the first three criteria, data is provided Anchors based on the smallest and largest expected values to be considered can be used to establish ranges, which are then used to convert measures into utilities Appropriate adjustments of signs can reflect measures to be maximized and minimized Transportation availability measures provided were subjective, without numeric values These can be transformed into utilities categorically A rating of great could be assigned a utility value of 1 0, a rating of good a utility value of 0 8, a rating of fair a utility value of 0 3, and a rating of poor a utility rating of 0 This would yield the final set of single-attribute utilities for the decision problem presented in Table 2

Table 2

Alternative	Cost	Growth	Skill	Transport
New York	0 175	0 8	0 6	1 0
Los Angeles	0 275	1 0	0 8	1 0
Phoenix	0 425	0 5	0 4	0 8
Houston	0 450	0 4	0 5	0 8
Denver	0 475	0 25	0 25	0 8
Dallas	0 5125	0 2	0 2	0 3
Nashville	0 550	0 1	0 1	0 0

3. DEVELOPMENT OF SMART WEIGHTS

The last required data is the set of relative weights for the four criteria The process of swing weighting would begin by considering the possible range of measures for all criteria, and asking in turn which criterion would be most important to move from its worst measure to its best measure (see Table 3)

Table 3

Criterion	Worst	Best
Cost ($ million)	$15 million	$7 million
Growth potential	200,000	600,000
Skilled labor availability	1000	4000
Transportation	Poor	Great

In this case, the decision-maker might think that moving cost from $15 million to $7 million was the most important of the four criteria Given that cost is the most important criterion, the remaining three criteria are considered in turn Moving growth potential from 200,000 to 600,000 might be considered as more important than the other two remaining criteria Finally, the last two criteria are compared in a similar manner For our purposes, we might assume that moving skilled labor availability from 1000 to 4000 was considered more important than moving transportation availability from poor to great These evaluations yield the rank order

$$w_{\text{cost}} > w_{\text{growth}} > w_{\text{skill}} > w_{\text{trans}}$$

The next step is to determine the relative weights This is done by asking the decision maker what the relative importance of moving the other three criteria from their worst to best measures would be if w_{cost} were 100 A possible response might be as presented in Table 4

Table 4

w_{cost}	100
w_{growth}	50
w_{skill}	25
w_{trans}	12

Table 5

w_{cost}	100/187	0 535
w_{growth}	50/187	0 267
w_{skill}	25/187	0 134
w_{trans}	12/187	0 064

This would yield a total of 187, which could then be divided into each of the measures to obtain a normalized set of weights that sum to 1 0, as shown in Table 5

The last step of the SMART method with swing-weighting is to apply the original model to calculate the weighted overall utility (value) for each alternative This consists of multiplying the criterion weight times the alternative's criterion utility over all four criteria, and summing The results for this example are presented in Table 6 These value functions allow ranking of the seven sites Los Angeles is a clear first choice, followed by Houston and Phoenix, which have almost identical value functions New York, while a dominated solution, is much preferred to Denver, Dallas, and Nashville

Table 6

Alternative	Cost	w_{cost}	Growth	w_{growth}	Skill	w_{skill}	Transp	w_{trans}	Value
New York	0 175	0 535	0 8	0 267	0 6	0 134	1 0	0 064	**0.452**
Los Angeles	0 275	0 535	1 0	0 267	0 8	0 134	1 0	0 064	**0.585**
Phoenix	0 425	0 535	0 5	0 267	0 4	0 134	0 8	0 064	**0.4657**
Houston	0 450	0 535	0 4	0 267	0 5	0 134	0 8	0 064	**0.4658**
Denver	0 475	0 535	0 25	0 267	0 25	0 134	0 8	0 064	**0.405**
Dallas	0 5125	0 535	0 2	0 267	0 2	0 134	0 3	0 064	**0.374**
Nashville	0 550	0 535	0 1	0 267	0 1	0 134	0 0	0 064	**0.334**

Weights can vary a great deal by decision-maker In this case, there was a moderate dispersion of weights We check two other cases following Kirkwood and Corner [13] one with all four criteria weighted equally, and one with greater dispersion in weights For the case with four equal weights over criteria, see Table 7

In this case, Los Angeles remains the first choice, but now the dominated site at New York is second in preference The relative order of the other five alternatives remains the same

A more diverse set of weights might assign $w_{\text{skill}} = 4w_{\text{trans}}$, $w_{\text{growth}} = 4w_{\text{skill}}$, and $w_{\text{cost}} = 4w_{\text{growth}}$, yielding a normalized set of weights shown in Table 8

These weights would yield value functions as shown in Table 9

Note that now the six nondominated alternatives are all very close in value, with Houston holding a slight edge over Los Angeles, Phoenix, Nashville, Dallas, and Denver in turn New

Table 7

Alternative	Cost	w_{cost}	Growth	w_{growth}	Skill	w_{skill}	Transp	w_{trans}	Value
New York	0 175	0 25	0 8	0 25	0 6	0 25	1 0	0 25	**0 644**
Los Angeles	0 275	0 25	1 0	0 25	0 8	0 25	1 0	0 25	**0 769**
Phoenix	0 425	0 25	0 5	0 25	0 4	0 25	0 8	0 25	**0 531**
Houston	0 450	0 25	0 4	0 25	0 5	0 25	0 8	0 25	**0 537**
Denver	0 475	0 25	0 25	0 25	0 25	0 25	0 8	0 25	**0 444**
Dallas	0 5125	0 25	0 2	0 25	0 2	0 25	0 3	0 25	**0 303**
Nashville	0 550	0 25	0 1	0 25	0 1	0 25	0 0	0 25	**0.187**

Table 8

w_{cost}	64/85	0 753
w_{growth}	16/85	0 188
w_{skill}	4/85	0 047
w_{trans}	1/85	0 012

Table 9

Alternative	Cost	w_{cost}	Growth	w_{growth}	Skill	w_{skill}	Transp	w_{trans}	Value
New York	0 175	0 753	0 8	0 188	0 6	0 047	1 0	0 012	**0 322**
Los Angeles	0 275	0 753	1 0	0 188	0 8	0 047	1 0	0 012	**0 445**
Phoenix	0 425	0 753	0 5	0 188	0 4	0 047	0 8	0 012	**0 442**
Houston	0 450	0 753	0 4	0 188	0 5	0 047	0 8	0 012	**0.447**
Denver	0 475	0 753	0 25	0 188	0 25	0 047	0 8	0 012	**0.426**
Dallas	0 5125	0 753	0 2	0 188	0 2	0 047	0 3	0 012	**0 436**
Nashville	0 550	0 753	0 1	0 188	0 1	0 047	0 0	0 012	**0 438**

York is a great deal worse in value than the other six sites With this set of very diverse weights, the relative ranking has been reversed

4. CENTROID

The centroid method is identical to the SMART method, with the exception that weights are assessed based on the rank order of criteria importance (considering scale) The centroid method assigns weights as follows, where w_1 is the weight of the most important objective, w_2 the weight of the second most important objective, and so on For k objectives,

$$w_1 = \frac{(1 + 1/2 + 1/3 + \quad + 1/k)}{k},$$

$$w_2 = \frac{(0 + 1/2 + 1/3 + \quad + 1/k)}{k},$$

$$w_k = \frac{(0 + 0 + \quad + 0 + 1/k)}{k}$$

The sum of these weights will equal 1 0 The more objectives that exist, the less error this approximation involves For two objectives, $w_1 = (1+1/2)/2 = 0\ 75$ and $w_2 = (0+1/2)/2 = 0\ 25$ While this would minimize the maximum error (weight extremes would be $w_1 = 1$ and $w_2 = 0$, $w_1 = 0\ 5$ and $w_2 = 0\ 5$), with only two objectives the error could be substantial With more objectives, the error for ranked objectives will be much less Single-attribute utilities for each criterion could be obtained in the same manner as with SMART

For this example, considering the range of possible performance levels, the rank order of the four criteria would be

Cost > Growth > Skill > Transportation

Weights would be estimated by finding the centroid, the mean of the four extreme points (see Table 10)

Table 10

Cost	Growth	Skill	Transport
1 0	0	0	0
0 5	0 5	0	0
0 333	0 333	0 333	0
0 25	0 25	0 25	0 25
0 52083	**0.27083**	**0.14583**	**0 0625**

These weights could be applied directly as in SMART (see Table 11) The rank order of alternatives obtained in this case are identical to those obtained with the initial weights in the SMART example

Table 11

Alternative	Cost	w_{cost}	Growth	w_{growth}	Skill	w_{skill}	Transp	w_{trans}	Value
New York	0 175	0 521	0 8	0 271	0 6	0 146	1 0	0 062	**0.458**
Los Angeles	0 275	0 521	1 0	0 271	0 8	0 146	1 0	0 062	**0 593**
Phoenix	0 425	0 521	0 5	0 271	0 4	0 146	0 8	0 062	**0.4649**
Houston	0 450	0 521	0 4	0 271	0 5	0 146	0 8	0 062	**0.4654**
Denver	0 475	0 521	0 25	0 271	0 25	0 146	0 8	0 062	**0.401**
Dallas	0 5125	0 521	0 2	0 271	0 2	0 146	0 3	0 062	**0 369**
Nashville	0 550	0 521	0 1	0 271	0 1	0 146	0 0	0 062	**0.328**

5. CONSIDERATION OF CONDITIONAL UTILITY RANGE

Up to this point, what we have presented has been done before by studies referenced We now extend this work by considering conditional utilities The analysis based on ordinal input can be carried one step further, by considering the possible ranges of conditional utilities on attributes For the seven alternatives under consideration (with A_1 representing New York, A_2 Los Angeles, etc), we denote

$U(A_i)$, $\forall i = 1$ to 7 is the overall utility for each alternative using formula (1) above,

$U_j(A_i)$, $\forall i = 1$ to 7, $\forall j = 1$ to 4 is the conditional utility of attribute j for alternative i,

subject to the following constraints

$$w_j \in [0,1], \qquad \Sigma_j w_j = 1 \tag{2}$$

We assume that the decision-maker has carried out the ordinal ranking of attributes (considering attribute scales) and reached the conclusion that attribute 1 is the most important, attribute 2 is second-in-importance, attribute 3 is third-in-importance, and attribute 4 is the least important (If otherwise, we could simply renumber the attributes, so in further consideration, the attribute with a smaller number will always be the attribute with a higher weight) Then, the permissible values of attribute weights are as follows

$$\begin{gathered} w_1 \in \left[\frac{1}{4}, 1\right], \qquad w_2 \in \left[0, \frac{1}{2}\right], \qquad w_3 \in \left[0, \frac{1}{3}\right], \qquad w_4 \in \left[0, \frac{1}{4}\right], \\ w_1 + w_2 + w_3 + w_4 = 1, \qquad w_1 \geq w_2 \geq w_3 \geq w_4 \end{gathered} \tag{3}$$

Expression (3) actually determines some bounded subset of weights obtained by the "narrowing" of an initial set (2) based on additional information about the ranking of attributes The extreme points for such a bounded subset of weights are

$$\{w_1 = 1,\ w_2 = 0,\ w_3 = 0,\ w_4 = 0\}, \qquad \left\{w_1 = \frac{1}{2},\ w_2 = \frac{1}{2},\ w_3 = 0,\ w_4 = 0\right\},$$
$$\left\{w_1 = \frac{1}{3},\ w_2 = \frac{1}{3},\ w_3 = \frac{1}{3},\ w_4 = 0\right\}, \qquad \left\{w_1 = \frac{1}{4},\ w_2 = \frac{1}{4},\ w_3 = \frac{1}{4},\ w_4 = \frac{1}{4}\right\}$$

Taking into consideration that the weights sum to one, and therefore,

$$w_4 = 1 - (w_1 + w_2 + w_3),$$

it is possible to geometrically represent this bounded subset of weights in three-dimensional space, where $A\{1,0,0\}$, $B\{1/2,1/2,0\}$, $C\{1/3,1/3,1/3\}$, and $D\{1/4,1/4,1/4\}$ are the vertices of a triangular pyramid In the three-dimensional space of weights $\{w_1, w_2, w_3\}$, each current point $\{w_{1T}, w_{2T}, w_{3T}\}$ within a triangular pyramid $ABCD$ represents a permissible combination of weights The overall utility (for currently analyzed alternative A_i), corresponding to this point, could be found as

$$U_T(A_i) = w_{1T} * u_{1i} + w_{2T} * u_{2i} + w_{3T} * u_{3i} + [1 - (w_{1T} + w_{2T} + w_{3T})] * u_{4i} \tag{4}$$

As is known from linear programming, a linear objective function attains its maximum and minimum values at the boundaries of a feasible region For the case under consideration, linear form (4) could achieve its maximum and minimum values in the vertices A, B, C, or D (or in case of multiple optimal solutions at sides or planes connecting the points in question)

Therefore, knowing the values of overall utilities in extreme points A, B, C, and D, we can also estimate the maximum and minimum level of overall utilities for any concrete values of conditional utilities u_{i1}, u_{i2}, u_{i3}, and u_{i4} for all possible weights consistent with the ordinal ranking of attributes, i e , for all possible weights determined by expression (4)

Therefore, let us first estimate

$$U_A = u_{i1}, \qquad U_B = \frac{u_{i1} + u_{i2}}{2},$$
$$U_C = \frac{u_{i1} + u_{i2} + u_{i3}}{3}, \qquad U_D = \frac{u_{i1} + u_{i2} + u_{i3} + u_{i4}}{4}$$

We label the entire interval of overall utilities covering all possible combinations of weights for the case of ordinal ranking as the "uncertainty interval" This corresponds to all points within

triangular pyramid $ABCD$ The term "uncertainty interval" reflects the uncertainty in overall utility caused by the lack of knowledge about the actual values of weights This interval may be represented on a numerical axis as a numeric interval $[U_L, U_H]$, where the interval's bounds U_L and U_H equal accordingly to maximum and minimum value of linear overall utility form (4) In other words, U_L and U_H are the values of current overall utility in the lowest and highest point of the uncertainty interval (in the extreme point with the highest current value of utility) for the alternative under analysis

In accordance with the above, we can write down the following expressions

$$U_H = \max\{U_A, U_B, U_C, U_D\}, \qquad U_L = \min\{U_A, U_B, U_C, U_D\} \tag{5}$$

We can also estimate the value of overall utility in the center of the uncertainty interval

$$U_M = \frac{U_H - U_L}{2} \tag{6}$$

The information about the estimated value may be very valuable to the decision maker from the point of view of both deeper understanding of an initial situation and ways this situation could be improved Let us demonstrate for the above example

First, let us calculate overall utilities using (5) and (6) for all alternatives

For Alternative New York

$$U_H = 0\ 6438, \qquad U_L = 0\ 1750, \qquad U_M = 0\ 4094$$

For Alternative Los Angeles

$$U_H = 0\ 7688, \qquad U_L = 0\ 2750, \qquad U_M = 0\ 5219$$

For Alternative Phoenix

$$U_H = 0\ 5313, \qquad U_L = 0\ 4250, \qquad U_M = 0\ 4782$$

For Alternative Houston

$$U_H = 0\ 5375, \qquad U_L = 0\ 4250, \qquad U_M = 0\ 4813$$

For Alternative Denver

$$U_H = 0\ 4750, \qquad U_L = 0\ 3250, \qquad U_M = 0\ 4000$$

For Alternative Dallas

$$U_H = 0\ 5125, \qquad U_L = 0\ 3031, \qquad U_M = 0\ 4078$$

Table 12

	New York	Los Angeles	Phoenix	Houston	Denver	Dallas	Nashville
A	0 175	0 275	0 425	0 450	0 475	0 512	**0 550**
B	0 487	**0 637**	0 462	0 425	0 362	0 356	0 325
C	0 525	**0 692**	0 442	0 450	0 325	0 304	0 250
D	0 644	**0 769**	0 531	0 537	0 444	0 303	0 187

For Alternative Nashville

$$U_H = 0\ 5500, \qquad U_L = 0\ 1875, \qquad U_M = 0\ 3688$$

Ranking alternatives by the value of U_M (the center of uncertainty interval) provides almost the same results as ranking by centroid points The only difference is the alternatives Denver and Dallas changed places See Table 12 At point A, Nashville is preferred At points B, C, and D, Los Angeles is preferred Note that New York is dominated by Los Angeles, but that the other six alternatives are all nondominated

Uncertainty intervals can be developed for each alternative These uncertainty intervals provide a means for a deeper understanding of the current situation Although alternative Los Angeles seems to be better than the others, it cannot be guaranteed that this alternative will be the best under all possible sets of weights satisfying the ordinal specification If a decision-maker is able to clearly choose among existing alternatives, there is no need in further analysis On the other hand, if there is some hesitation, our approach proposes ways for improvement of a current alternative See Table 13

Table 13

	New York	Los Angeles	Phoenix	Houston	Denver	Dallas	Nashville
U_H	0 644	0 769	0 531	0 537	0 475	0 512	0 550
U_M	0 409	0 522	0 478	0 481	0 400	0 408	0 369
U_L	0 175	0 275	0 425	0 425	0 325	0 303	0 187

Let us assume that decision-maker wishes to improve alternative Los Angeles in such a way that it will become absolutely the best choice Following linear programming sensitivity analysis, consider changing only one of the values of conditional utilities u_{21}, u_{22}, u_{23}, and u_{24} We eliminate alternative New York, as it is dominated by Los Angeles From analysis of the uncertainty interval for alternative Los Angeles, we can make two important conclusions The first is positive—after elimination of New York, points D, C, and B for alternative Los Angeles are located higher than the highest points of the uncertainty intervals for all remaining alternatives Therefore, only point A needs to be "raised" to provide an absolute dominance for alternative Los Angeles (points B, C, and D will never "descend", since, in accordance with (1), they are the increasing functions of conditional utilities)

The second conclusion is negative—since the lowest point of the uncertainty interval is point A, it may be "raised" only by the way of increase in conditional utility u_{21} ($U_{1A} = u_{21}$) Therefore, no improvements in growth, skill, and transport will make alternative Los Angeles absolutely better than the other five alternatives The only way to ensure strict dominance for alternative Los Angeles is to cut its cost

For the currently analyzed alternative, let us designate $U_{\max}$ the value of maximum possible overall utility for all other alternatives (the highest point of all uncertainty intervals for all alternatives excluding the current one) After the preliminary elimination of dominated alternative

New York, alternative Nashville will have the greatest value of U_H among all remaining alternatives (excluding the current alternative Los Angeles) Therefore, $U_{\max} = U_H$ (for Nashville) $= 0\ 5500$ Although alternative Nashville was ranked last, this alternative is the toughest for Los Angeles to dominate This is because the rank order of extreme points for Nashville is opposite to that of Los Angeles

The increase Δu_1 of conditional utility u_1, necessary to provide an absolute dominance of alternative Los Angeles over all the other ones, may be found from the expression

$$U_{A0} + \Delta u_1 > U_{\max} \tag{7}$$

Therefore, the minimum required value of this increase Δu_{1G} is determined as follows

$$\Delta u_{1G} = U_{\max} - U_{A0} = 0\ 5500 - 0\ 2750 = 0\ 2750 \tag{8}$$

For the newly generated alternative, we calculate the overall utilities at the extreme points

$$\begin{aligned}
U_A &= U_{\max} = 0\ 5500, \\
U_B &= U_{B0} + \frac{\Delta u_1}{2} = 0\ 6375 + \frac{0\ 2750}{2} = 0\ 7750, \\
U_C &= U_{C0} + \frac{\Delta u_1}{3} = 0\ 6917 + \frac{0\ 2750}{2} = 0\ 7834, \\
U_D &= U_{D0} + \frac{\Delta u_1}{4} = 0\ 7688 + \frac{0\ 2750}{2} = 0\ 8376
\end{aligned}$$

After this improvement, alternative Los Angeles would have the level of utility equal to 0 5500 at the lowest point of its uncertainty interval This will provide a strict dominance over all the other alternatives

We can also use the formulation to calculate

$$\text{utility} = 1 - \left[\frac{(\text{cost} - 7)}{8}\right], \qquad \text{the corresponding required value of cost,}$$

$$\text{cost} = 15 - 8 * \text{utility} = 15 - 8 * 0\ 5500 = \$10\ 6 \text{ million}$$

If alternative Los Angeles has the value of cost 10 6 million dollars (as, for example, alternative Nashville), it will become absolutely the best choice

This case involved limited specific changes The number of changes, in general, can involve a number of degrees of freedom For instance, in the case of the Nashville site, for weight set A it is the preferred choice (it has the lowest cost) However, it is the poorest performer on each of the other three criteria For the other extreme points, the required improvement could come from any of these three criteria (or for that matter, improving the Nashville site's cost even more)

At weight set B, there is a 0 5 weight for both cost and growth Either or both of those measures could be improved for the Nashville site, such that the overall value for this site would equal or exceed the highest other alternative for this weight set (the Los Angeles site, with a value of $(0\ 5 \times 0\ 275) + (0\ 5 \times 1\ 0) = 0\ 6375$) Moving on to weight set $C(1/3, 1/3, 1/3, 0)$, Nashville currently has a value calculation of 0 250, which is the worst of all alternatives The best alternative score at this set of weights is for Los Angeles, at 0 692 Improvement on no single criterion would be sufficient to make Nashville have as high a score as Los Angeles An infinite number of combinations of improvement would Similar results occur for weight set D $(1/4, 1/4, 1/4, 1/4)$

6. CONCLUSIONS

The present article's main objective is to elaborate a new approach extending estimation of a centroid point for the product of weight times utility A secondary purpose is to use this framework to show how sensitivity analysis could be conducted

Elicitation of weights is usually a time consuming process and is often controversial, as well It is difficult to derive exact weights, and it is also difficult to determine consistent boundaries for the intervals within which actual weights are located In such circumstances, ordinal ranking could be a reasonable compromise that uses input of consistent information and often provides output rank order of alternatives similar to the rank order based upon the use of cardinal information

The proposed approach provides the decision-maker with more information about the degree and sources of uncertainty with respect to a preferred solution It adds minimization of the maximum error by estimating the value of an overall utility in the center of the uncertainty interval It also provides the decision-maker with information about the positions of all extreme points for all competing alternatives If a decision maker is hesitant about choosing among existing alternatives, the proposed approach allows determination of which improvements in the values of existing alternatives' parameters will result in increasing its performance to the level where this alternative becomes obviously preferable to all the other alternatives

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