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A comparison of stochastic dominance and stochastic DEA for vendor evaluation

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Vendor selection involves decisions balancing a number of conflicting criteria. Data envelopment analysis (DEA) is a mathematical programming approach capable of identifying non-dominated solutions, as well as assessing relative efficiency of dominated solutions. A simple multi-attribute utility function can be applied to a small set of alternatives, providing a tool to assess relative value, but is subject to error if estimated measures are not precise. This paper compares stochastic DEA with a multiple-criteria model in a vendor selection model involving multiple criteria, reporting simulation experiments varying the degree of uncertainty involved in model parameters.

Keywords: Vendor selection; Stochastic data envelopment analysis; Multiple criteria; Simulation

1. Introduction

Vendor evaluation is a very important operational decision. Important decisions include which vendors to employ and quantities to order from each vendor. With the increase in outsourcing and the opportunities provided by electronic business to tap worldwide markets, these decisions are becoming ever more complex. The presence of multiple criteria in these decisions has long been recognized. Dickson (1996) identified 23 distinct criteria in various vendor selection problems. Weber *et al.* (1991) found multiple criteria in 47 of the 76 vendor selection articles that they reviewed. Table 1 compares criteria used in 12 studies over the period 1996 through 2006 with the top row containing the number of the 76 articles Weber *et al.* reviewed that included the same criteria. Price, quality, and response have become endemic.

In the past, more emphasis seems to have been placed on managerial and organizational reputation, expertise, and attitude. Some criteria continue to have moderate presence, such as the availability of adequate facilities, technological innovation, and the ability to provide service. More recent articles place an increased emphasis on flexibility and agility, probably reflecting the increased use of e-commerce, rapid delivery, and more responsive delivery.

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Vendor selection decisions often require tradeoffs among a number of criteria. Furthermore, data on specific vendor performance is imprecise of necessity. Some studies (Kulak and Kahraman 2005, Chang *et al.* 2006a) have recognized this imprecision through methods accommodating fuzzy data, but most methods fail to consider uncertainty. Therefore, we compare existing stochastic dominance with stochastic data envelopment analysis (DEA).

Stochastic dominance analysis has been presented in the past. This paper applies simulation to provide a more tractable solution approach with the ability to give more detail in a straightforward manner than stochastic dominance analysis. Thus we are not presenting a new stochastic dominance model, but rather use simulation as a way to implement one reported in the literature. These results are compared with DEA. Our purpose is not to compare stochastic dominance models, but rather to demonstrate how DEA models compare. Section 2 of the paper reviews vendor selection methodologies. Section 3 gives the vendor selection model used, with results. Section 4 compares results, and section 5 provides conclusions.

Table 1. Criteria used in prior studies.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
Weber *et al.* (1991) – 74 prior	61	40	44	7	23	15			2	7			10	7	8
Patton (1996)	x	x	x	x	x									x	
Petroni and Braglia (2000)	x	x	x		x	x							x		
Tam and Tummala (2001)	6	x	x			9	x								2
Chan (2003)	x	x	x			x	x								
Wang *et al.* (2004)	3		4		3		2								
Zhu (2004)	3	x	2	2	x										
Liu and Hai (2005)	x	2	2		3	2						2	2		
Ounnar and Pujo (2005)	x	x	x	x	x	x	x			X	x				
Tseng and Lin (2005)		x				x	x		x						
Kulak and Kahraman (2005)	x	x	x				x	x							
Chang *et al.* (2006b)	x	x	x	x		x									
Talluri *et al.* (2006)	x	x	x												

C1 Price/cost
C2 Acceptance/quality
C3 On-time response/logistics
C4 Service
C5 Production facilities/assets
C6 R&E in technology/innovation/design capability
C7 Flexibility/agility.
C8 Documentation
C9 Communication
C10 Performance
C11 Strategic
C12 Discipline
C13 Management and organization
C14 Financial
C15 Expertise/reputation.
X – indicates presence.
Numbers indicate the number of specific criteria in this general area.

2. Vendor selection methodologies

Many methods have been proposed to aid vendor selection. Talluri *et al*. (2006) listed no fewer than 14 different methodologies over the period 1969 through 2003 (including multiple-criteria methods and DEA). Kulak and Kahraman (2005) came up with a different categorization of five methods: profile and checklist methods, scoring methods, cost-benefit analysis, mathematical programming, and fuzzy analysis. DEA models have been presented by Kleinsorge *et al*. (1992), Weber and Desai (1996), Weber *et al*. (1998), Liu *et al*. (2000), Narasimhan *et al*. (2001), and Talluri (2002).

2.1 *Vendor selection procedure*

The stochastic DEA and stochastic dominance model applied through simulation are used to measure vendor efficiency. The model aims to maximize the efficiency of vendor subject to attaining desired quality, price, performance, facilities/capabilities levels. In our stochastic DEA model, all the attributes are deemed as outputs since they are normalized as in Moskowitz *et al*. (2000). The process is to (1) identify criteria; (2) identify alternative vendors; (3) select measures; and (4) use a model to rank-order vendors. Our focus will be on measures that are as objective as possible, to include uncertain elements. Alternative models considered in this paper are stochastic dominance, simulation, and stochastic DEA.

2.2 *Stochastic vendor selection*

Moskowitz *et al*. (2000) gave a multiple criteria vendor evaluation model consisting of the hierarchical structure given in table 2.

The overall categories are quality, price, performance, and facilities/capabilities. Each of these four categories has two to four criteria reflecting the performance of various vendors that were available. In the Moskowitz *et al*. case, there were nine

Table 2. Criteria hierarchy (Moskowitz *et al*. 2000).

Overall	Primary categories	Criteria
Vendor evaluation	Quality	1. Quality personnel
		2. Quality procedure
		3. Concern for quality
		4. Company history
	Price	5. Price of Quality
		6. Actual price (negotiated or quoted)
		7. Financial ability
	Performance	8. Technical
		9. Delivery history
		10. Technical assistance
	Facilities/capabilities	11. Production capability
		12. Manufacturing equipment

Table 3. Means and standard deviations (Moskowitz *et al.* 2000).

Criteria	*V*1	*V*2	*V*3	*V*4	*V*5	*V*6	*V*7	*V*8	*V*9
1 Quality personnel	85* (5.2)**	82 (4.2)	90 (3.1)	78 (12.8)	95 (1.5)	75 (2.9)	90 (1.7)	70 (12.2)	75 (2.8)
2 Quality procedure	80 (3.3)	88 (4.2)	85 (5.1)	90 (4.2)	75 (5.6)	82 (2.2)	82 (4.2)	90 (33)	78 (3.8)
3 Concern for quality	80 (4.7)	83 (5.5)	70 (5.5)	75 (14.3)	85 (5.8)	85 (1.9)	75 (5.9)	90 (2.4)	90 (1.1)
4 Company history	90 (5.5)	88 (4.5)	75 (7.0)	85 (5.6)	70 (5.6)	80 (4.1)	80 (4.6)	85 (4.5)	82 (3.7)
5 Price-quality	90 (6.0)	88 (5.1)	95 (3.2)	78 (6.1)	80 (9.2)	85 (1.7)	82 (5.1)	90 (3.1)	85 (2.2)
6 Actual price	73 (3.9)	78 (6.4)	68 (6.5)	80 (5.8)	80 (8.8)	78 (3.2)	85 (2.1)	80 (5.3)	83 (2.7)
7 Financial ability	85 (4.4)	92 (4.8)	74 (5.0)	85 (6.5)	78 (6.6)	90 (0.8)	80 (3.9)	75 (5.1)	70 (3.7)
8 Technical performance	70 (5.3)	80 (5.0)	90 (3.4)	60 (9.0)	100 (0.0)	80 (1.8)	70 (6.2)	78 (7.7)	82 (2.5)
9 Delivery history	80 (4.1)	75 (5.5)	73 (5.1)	85 (6.0)	70 (7.2)	70 (2.5)	73 (3.3)	85 (5.0)	80 (3.1)
10 Technical assistance	55 (5.3)	55 (5.1)	78 (5.6)	65 (8.2)	75 (12.2)	78 (2.2)	72 (3.6)	68 (9.9)	76 (4.2)
11 Production capability	85 (4.0)	80 (6.6)	78 (5.1)	65 (7.3)	55 (8.8)	80 (2.3)	75 (3.3)	83 (2.9)	78 (2.3)
12 Manufacturing equipment	70 (5.6)	83 (4.7)	90 (2.1)	95 (2.2)	100 (0.0)	77 (1.8)	85 (2.4)	79 (4.2)	70 (2.3)

* = means; ** = standard deviations.

vendors, for which relative performance scores were available. Table 3 gives means (and standard deviations) for each score.

Stochastic dominance assumes correlated variance. Moskowitz *et al.* (2000) considered two sets of weights on criteria. Random weights yielded four alternatives as first-order stochastically non-dominated (*V*2, *V*4, *V*6, and *V*8) in their study. Moskowitz *et al.* then applied ordinal weights, using the order of importance as:

$$W_2 > W_1 > W_3 > W_4 > W_5 > W_6 > W_7 > W_{10} > W_8 > W_9 > W_{11} > W_{12}$$

This represents one possible order that decision-makers might express in light of their preferences. This set of ordinal weights yielded rank orders V_2 dominating V_8, which dominated the other seven alternatives. Thus more knowledge about weights can yield more complete rankings of alternatives. However, even greater clarity might be obtainable through use of alternate methods, such as DEA or simulation.

3. Multi-criteria model with simulation

Simulation of multiple criteria models is now easily accomplished, using such tools as crystal ball, which supports spreadsheets such as EXCEL (Evans and Olson 2005). Simulation can replicate the results of stochastic dominance by assuming a set of weights with ranges and order as specified. Selection is identified by calculating the simulated value function for each of the nine vendors, with the highest value function selected. If enough simulation runs are made, it can reflect any complexities that might be present in a model. Simulation has been applied in fuzzy data mining models (Olson and Wu 2006). The simple multi-attribute rating theory (SMART - Edwards and Barron 1994) model simply bases selection on the rank order of the product of criteria weights and alternative scores over these criteria. The data given in table 2 can directly be applied in a Crystal Ball model. Using random weights and controlling for random scores (so that equal luck is given to each alternative over each criterion), stochastically non-dominated solutions are the only ones with the possibility of having the greatest score. This in fact was attained in our model. The ordinal weights suggested by Moskowitz *et al.* (2000) were also applied. Table 4 gives the proportion of 1000 simulation runs, yielding probabilities of each alternative being preferred.

The equal weight model confirmed the stochastic dominance results (we got the same thing that Moskowitz *et al.* got). However, the simulated multi-attribute model yields more information, showing the probabilities of each alternative vendor being preferred for the data given. Adding more information about relative weights will provide yet more information, as it should. Here, the most probable selection under conditions of random weights with equal probabilities was never selected, as the weights associated with this vendor's strengths were given relatively low importance. While Moskowitz *et al.* identified *V*2 and *V*6, table 4 shows through simulation results that *V*4 was also non-dominated with this set of ordinal weights. Vendor alternative *V*2 turned out to be the most probable best choice for the ordinal weights given.

Table 4. Weights obtained from simulation.

	$V1$	$V2$	$V3$	$V4$	$V5$	$V6$	$V7$	$V8$	$V9$
Equal weights	0	0.03	0	0.08	0	0.36	0	0.53	0
Ordinal weights	0	0.71	0	0.22	0	0.07	0	0	0

4. DEA analysis

Charnes *et al.* (1978) first introduced DEA (CCR) for efficiency analysis of Decision-making Units (DMU). DEA can be used for modeling operational processes, and its empirical orientation and absence of *a priori* assumptions have resulted in its use in a number of studies involving efficient frontier estimation in both nonprofit and in private sectors. DEA has become a leading approach for efficiency analysis in many fields, such as supply chain management (Ross and Droge 2002), business research and development (Verma and Sinha 2002), petroleum distribution system design (Ross and Droge 2004), military logistics (Sun 2004), and government services (Narasimhan *et al.* 2005). DEA and multi-criteria decision-making models have been compared and extended (Lahdelma and Salminen 2006). Section 4.1 develops stochastic DEA vendor evaluation model since stochastic DEA takes into account the risk and uncertainty embedded in the business environment.

4.1 *Stochastic DEA mathematical formulation*

Stochastic DEA constructs production frontiers that incorporate both inefficiency and stochastic error. The stochastic DEA frontier associates extreme outliers with the stochastic error term and this has the effect of moving the frontier closer to the bulk of the producing units. As a result, the measured technical efficiency of every DMU is raised relative to the deterministic model. In some realizations, some DMUs will have a super-efficiency larger than unity (Olesen and Petersen 1995, Cooper *et al.* 1996, Cooper *et al.* 2002).

Now we consider the stochastic vendor selection model. Consider N suppliers to be evaluated; each has s random variables. Note that all input variables are transformed to output variables, as was done in Moskowitz *et al.* (2000). Classical DEA does not apply in our case since it asks for a classification of attributes into input and output variables and we only have transformed "output variables." The variables of supplier j $(j=1,2,\ldots,N)$ exhibit random behavior represented by $\tilde{y}_j=(\tilde{y}_{1j},\ldots,\tilde{y}_{sj})$, where each $\tilde{y}_{rj}(r=1,2,\ldots,s)$ has a known probability distribution. By maximizing the expected efficiency of a vendor under evaluation, the following model (1) is developed.

$$\begin{aligned} \max_{v} \quad & E(v^T\tilde{y}_0) \\ \text{s.t.} \quad & \Pr(v^T\tilde{y}_j \le \beta_j) \ge 1-\alpha_j, \quad j=1,2\cdots N \\ & v \ge 0 \end{aligned} \tag{1}$$

In (1), E refers to the expectation and Pr stands for probability. The first inequality constraint in (1) indicates that the efficiency score of supplier j is less than or equal to β_j, which represents the expected efficiency level of the jth supplier and is thus a prescribed value between 0 and 1. The v represents the virtual multipliers (weights) to be determined by solving the above problem and $\tilde{y}_0$ is the random output vector for the N vendors. The objective expected efficiency of (1) is interpreted as an "aspiration level" imposed by an outside authority and/or a budgetary limitation (Stedry 1962). The α_j $(0 \le \alpha_j \le 1)$ in the constraints are predetermined scalars which stand for an allowable risk of violating the associated constraints, where $1-\alpha_j$ indicates the probability of attaining the requirement. When $\alpha_j = 0$ in (1), it is certainly required that the output/input ratio becomes less than or equal to β_j Conversely, $\alpha_j = 1$ omits the requirement under any selection of weight multipliers. Note that our stochastic DEA model only considers a sort of output data, which differs from classical stochastic DEA where both input and output variables are considered. Thus computational complexity can be reduced.

To transform the stochastic DEA model (1) into a deterministic DEA, Charnes and Cooper (1959) (also see Huang and Li 2001) employed chance constrained programming (CCP, Charnes *et al.* 1958). The transformation steps presented in this study follow this technique and can be considered as a special case of their stochastic DEA (Cooper *et al.* 1999), where both stochastic inputs and outputs are used.

To proceed, we rewrite the 2nd constraints in model (1) as:

$$\Pr\left(\frac{v^T(\tilde{y}_j - \bar{y}_j)}{\sqrt{\mathrm{Var}_j}} \le \frac{\beta_j - v^T\bar{y}_j}{\sqrt{\mathrm{Var}_j}}\right) \ge 1 - \alpha_j \tag{2}$$

where $\mathrm{Var}_j = (v_1, v_2 \ldots v_s) \times$

$$\begin{pmatrix} \mathrm{Var}(\tilde{y}_{1j}) & (\tilde{y}_{1j}, \tilde{y}_{2j}), & \ldots, & \mathrm{Cov}(\tilde{y}_{1j}, \tilde{y}_{sj}) \\ \mathrm{Cov}(\tilde{y}_{2j}, \tilde{y}_{1j}) & \mathrm{Var}(\tilde{y}_{2j}), & \ldots, & \mathrm{Cov}(\tilde{y}_{2j}, \tilde{y}_{sj}) \\ \vdots & & \ddots & \vdots \\ \mathrm{Cov}(\tilde{y}_{sj}, \tilde{y}_{1j}), & \ldots & \ldots, & \mathrm{Var}(\tilde{y}_{sj}) \end{pmatrix} \times (v1, v2 \ldots vs)^T$$

Here Var_j indicates the variance-covariance matrix of the jth supplier in which the symbol "Var" stands for a variance and the symbol "Cov" refers to a covariance operator.

By CCP, a new variable following the standard normal distribution with zero mean and unit variance could be introduced as follows:

$$\tilde{z}_j = \frac{v^T(\tilde{y}_j - \bar{y}_j)}{\sqrt{\mathrm{Var}_j}}, \quad j = 1, \ldots, N. \tag{3}$$

Substitute expressions in (2) with (3):

$$\Pr\left(\tilde{z}_j \le \frac{\beta_j - v^T\bar{y}_j}{\sqrt{\mathrm{Var}_j}}\right) \ge 1 - \alpha_j \tag{4}$$

After a simple transformation we have (5) as:

$$\frac{\beta_j - v^T \bar{y}_j}{\sqrt{\mathrm{Var}_j}} \geq \Phi^{-1}(1-\alpha_j), \quad j = 1, \ldots, N. \tag{5}$$

where Φ stands for a cumulative distribution function of the normal distribution and Φ^{-1} indicates its inverse function. This completes the transformation of the probabilistic version of the linear output constraint into a deterministic non-linear form using what Charnes and Cooper (1959) refer to as a modified certainty equivalent.

Based on (5), model (1) can be written as the following (6):

$$\begin{aligned} \max_{v} \quad & E(v^T \tilde{y}_0) \\ \text{s.t.} \quad & \beta_j - v^T \bar{y}_j \geq \sqrt{\mathrm{Var}_j}\Phi^{-1}(1-\alpha_j) \\ & v \geq 0 \end{aligned} \tag{6}$$

This is a non-linear programming problem in the variables v, which faces computational difficulties due to the objective function and the constraints, including the variance $\sqrt{\mathrm{Var}_j}$, with quadratic expressions. To further reduce the model (6), we assume that $\tilde{y}_j$ follows a normal distribution $N(\bar{y}_j, B_j)$, where $\bar{y}_j$ is its vector of expected value and B_j indicates the variance-covariance matrix of the jth DMU by the following formula.

$$B_j = \begin{pmatrix} {}^j b_{11}^2 & {}^j b_{12}^2, & \ldots, & {}^j b_{1s}^2 \\ {}^j b_{21}^2 & {}^j b_{22}^2, & \ldots, & {}^j b_{2s}^2 \\ \vdots & & \ddots & \vdots \\ {}^j b_{s1}^2, & \ldots & \ldots, & {}^j b_{ss}^2 \end{pmatrix} \tag{7}$$

${}^j b_{rt}^2$ (i, $t = 1, 2, \ldots, s$) denotes the variance value of $\tilde{y}_{rj}$ if $r = t$ and its covariance value if $r \neq t$ of the jth DMU. In this case, we have

$$\mathrm{Var}_j = (v_1, v_2, \ldots, v_s) \times \begin{pmatrix} {}^j b_{11}^2 & {}^j b_{12}^2, & \ldots, & {}^j b_{1s}^2 \\ {}^j b_{21}^2 & {}^j b_{22}^2, & \ldots, & {}^j b_{2s}^2 \\ \vdots & & \ddots & \vdots \\ {}^j b_{s1}^2, & \ldots & \ldots, & {}^j b_{ss}^2 \end{pmatrix} \times (v_1, v_2, \ldots, v_s)^T = \left(\sum v_r^j b_{rr}\right)^2$$

$$(r = 1, \ldots, s; \; j = 1, \ldots, N) \tag{8}$$

Then model (8) can be reformulated as the following equivalent linear programming.

$$\max_{v} \quad v^T \bar{y}_0 \tag{9.1}$$

$$\text{s.t.} \quad \beta_j - v^T \bar{y}_j \geq v^T b_j \Phi^{-1}(1-\alpha_j), \quad j = 1, \ldots, N \tag{9.2}$$

$$v^T \geq 0 \tag{9.3}$$

where $b_j = ({}^j b_{rr}^2)_{s\times 1}$ is a s-dimension vector and $\bar{y}_0$ in the objective function denotes the mean output value of the 0th supplier ($0 \in [1, \ldots, N]$).

4.2 *Stochastic DEA result*

With data in table 3, we run model (9) by choosing different combinations of parameter $\alpha_j = \alpha$ and $\beta_j = \beta$ for all $j = 1, \ldots, N$. Thus there are two parameters that are not part of the given data set: α and β. We run the DEA model given in with various values for these parameters to see the sensitivity of the results. DEA stochastic efficiency scores of the nine vendors are obtained by running model (13) with different combinations between $\alpha \in \{0.05,\ 0.1,\ 0.2\}$ and $\beta \in \{0.85,\ 0.9\}$. The stochastic DEA model is solved nine times, each for one of the alternatives under evaluation. The formulation to compute the efficiency θ_1 for the first vendor is:

$$\begin{aligned} \text{Max } \theta_1 &= 85v_1 + 80v_2 + 80v_3 + 90v_4 + 90v_5 + 73v_6 + 85v_7 \\ &\quad + 70v_8 + 80v_9 + 55v_{10} + 85v_{11} + 70v_{12} \end{aligned}$$

Subject to: constraint for all vendor j from 1 to 9

$$v_i = 0 \text{ for all } i \text{ from 1 to 12}$$

v_i is defined in model (1) and denote the weight attached to the ith attribute. For $j = 2$, the constraint is specifically expressed as:

$$v^T(b_2\Phi^{-1}(1-\alpha) + \bar{y}_2) \le \beta.$$

Specifically,

$$\begin{aligned} &(82 + 4.2 * \Phi^{-1}(1-0.2))v_1 + (88 + 4.2 * \Phi^{-1}(1-0.2))v_2 \\ &\quad + (83 + 5.5 * \Phi^{-1}(1-0.2))v_3 + (88 + 4.5 * \Phi^{-1}(1-0.2))v_4 \\ &\quad + (88 + 5.1 * \Phi^{-1}(1-0.2))v_5 + (78 + 6.4 * \Phi^{-1}(1-0.2))v_6 \\ &\quad + (92 + 4.8 * \Phi^{-1}(1-0.2))v_7 + (80 + 5.0 * \Phi^{-1}(1-0.2))v_8 \\ &\quad + (75 + 5.5 * \Phi^{-1}(1-0.2))v_9 + (55 + 5.1 * \Phi^{-1}(1-0.2))v_{10} \\ &\quad + (80 + 6.6 * \Phi^{-1}(1-0.2))v_{11} + (83 + 4.7 * \Phi^{-1}(1-0.2))v_{12} \le 0.9 \end{aligned}$$

Table 5. Relative efficiency from stochastic DEA.

	*V*1	*V*2	*V*3	*V*4	*V*5	*V*6	*V*7	*V*8	*V*9	Average
*V*1	**95.40**	94.33	93.58	94.62	75.11	95.33	95.16	94.32	89.72	91.95
*V*2	93.56	**95.60**	94.63	95.02	79.37	93.93	94.53	92.15	90.02	92.09
*V*3	94.98	85.17	**95.37**	92.27	94.83	88.55	92.96	94.71	92.25	92.34
*V*4	89.61	90.28	95.93	**98.11**	89.23	93.88	94.32	97.45	94.88	93.74
*V*5	85.86	83.01	91.04	95.63	**98.10**	83.64	88.80	91.08	86.16	89.26
*V*6	92.69	92.87	92.84	92.11	92.32	**93.47**	93.29	88.46	92.86	92.32
*V*7	90.69	93.21	93.87	93.37	92.97	91.37	**93.96**	93.17	91.24	92.65
*V*8	94.93	94.40	96.30	94.66	86.37	94.29	94.15	**96.64**	95.97	**94.19**
*V*9	93.06	92.88	93.55	92.94	92.82	93.76	93.65	93.22	**93.78**	93.30

Table 6. Relative efficiency from stochastic DEA.

	$V1$	$V2$	$V3$	$V4$	$V5$	$V6$	$V7$	$V8$	$V9$	Average
$V1$	**93.43**	92.88	92.68	86.98	91.41	91.40	92.13	90.72	92.65	91.59
$V2$	94.79	**94.90**	94.79	93.81	94.58	94.58	94.64	94.62	94.78	94.25
$V3$	92.68	91.92	**94.59**	92.37	93.58	94.52	94.02	92.71	91.90	93.32
$V4$	90.00	90.93	91.29	**94.99**	90.60	90.81	92.16	92.52	91.28	91.99
$V5$	90.93	91.69	92.92	84.04	**95.00**	94.94	93.88	91.48	92.88	92.10
$V6$	92.33	92.38	92.32	87.18	92.26	**92.29**	92.33	92.21	92.40	91.72
$V7$	91.38	92.31	93.17	89.62	92.83	93.25	**93.60**	91.37	92.16	92.23
$V8$	94.71	94.65	93.64	93.68	92.80	92.23	92.96	**95.12**	94.53	93.54
$V9$	92.36	92.72	91.43	83.52	91.10	90.59	91.00	91.13	**92.80**	90.87

Efficiency value increases as the value of β goes up and the value of α decreases. This systematic trend is consistent with that in Sueyoshi (2000). Following Sueyoshi (2000), we fix $\alpha = 0.2$ and $\beta = 0.9$ for the rest of the paper. Moskowitz *et al.* (2000) assumed two types of weights: random weights and ordinal weights. To take into account these weight differences, we run our stochastic DEA in two cases: normal DEA in model (9) and model (9) coupled with weight restriction as identified in section 2.2, i.e. the following model (10).

$$\begin{array}{lll} \max\limits_{v} & v^T \bar{y}_0 & \\ \text{s.t.} & v^T & \text{in (9.2) and (9.3)} \\ & v^T & \text{in Moskowitz } et\ al. \text{ (2000).} \end{array} \tag{10}$$

Table 5 documents the stochastic efficiency for each vendor with parameter $\alpha = 0.2$, $\beta = 0.9$ and without restricting the weight, while table 6 documents the stochastic efficiency for each vendor by running stochastic DEA with restricted weight. Note that in tables 5 and 6 we also provide the cross efficiency for each vendor. The cross efficiency of the kth vendor is defined as the efficiency calculated using the weight optimized for another vendor j (Sexton *et al.* 1986). For example, after computing the stochastic efficiency for the first vendor, we obtain the optimal stochastic efficiency 0.954 and optimal weight vector

$$(0.0004, 0, 0, 0.0050, 0.0040, 0, 0, 0, 0, 0.00194, 0, 0).$$

The cross efficiency of the second vendor is computed as $0.0004*82 + 0*88 + 0*83 + 0.005*88 + 0.004*88 + 0*78 + 0*92 + 0*80 + 0*75 + 0.00194*55 + 0*80 + 0*83 = 93.56\%$, as documented in the cell of the third row and the second column in table 5. The cross efficiency can be deemed as an efficiency evaluation for alternative under-evaluation from the perspective of other alternatives, i.e. cross evaluation. The average of the cross efficiencies and DEA efficiency integrates self-evaluated score and cross-evaluated score (following Doyle and Green 1993) and thus provides another way to determine the non-dominated alternative(s). For example, in table 5, V_8 is the maximum in columns *V3* and *V9*, indicating that *V8* is identified as most efficient by *V3* and *V9*; similarly, *V1* is the maximum in both columns *V6* and *V7*; *V4* is the maximum in column *V8*. This means that *V1* and *V4*

Table 7. DEA results for each variable.

	CCR without weight restriction	Super CCR without weight restriction	CCR with weight restriction	Super CCR with weight restriction
*V*1	1.000	1.057	0.991	0.991
***V*2**	**1.000**	1.060	1.000	1.034
*V*3	1.000	1.108	1.000	1.020
***V*4**	**1.000**	1.087	1.000	1.013
*V*5	1.000	1.143	1.000	1.008
***V*6**	**1.000**	1.109	0.999	0.999
*V*7	1.000	1.062	1.000	1.000
***V*8**	**1.000**	1.074	1.000	1.017
*V*9	1.000	1.042	0.989	0.989

also have potential to be termed as non-dominated alternatives based on a cross-evaluation scheme. Below we will consider both the stochastic DEA efficiency and average stochastic cross efficiency.

In tables 5 and 6, stochastic efficiency for each vendor by solving nine linear programs is highlighted in bold. The last column shows the average stochastic cross efficiencies. Without restricting the weights, *V4* is identified as the most preferred vendor with a stochastic efficiency of 0.9811 at the aspiration level of 0.9 ($\beta = \mathbf{0.9}$) and a twenty percent allowable chance (risk, $\alpha = \mathbf{0.2}$) of failing to satisfy the constraint with which it is associated.

The ranking orders are as follows:

Stochastic efficiency without weight restriction using the diagonal:

$$V4 \succ V5 \succ V8 \succ V2 >\succ V1 \succ V3 \succ V7 \succ V9 \succ V6$$

where the symbol "$\succ$" denotes "is superior to."

Stochastic cross efficiency without weight restriction using averages:

$$V8 \succ V4 \succ V9 \succ V7 \succ V3 \succ V6 \succ V2 \succ V1 \succ V5$$

From table 6, we can see that the ranking orders are:

Stochastic efficiency with weight restriction:

$$V8 \succ V5 \succ V4 \succ V2 \succ V3 \succ V7 \succ V1 \succ V6 \succ V9$$

Stochastic cross efficiency with weight restriction:

$$V2 \succ V8 \succ V3 \succ V7 \succ V5 \succ V4 \succ V6 \succ V1 \succ V9$$

As can be expected, different approaches identify different non-dominated vendor. The same problem occurs in Moskowitz *et al*. (2000) where they identified completely different non-dominated vendors by using random weight assumption and ordinal weight assumption. Under assumption of random weights, Moskowitz *et al*. (2000) identified *V*6 and *V*8 as non-dominated vendors. The first order dominated vendors are *V*1, *V*3, *V*4, *V*7, *V*9.

Under assumption of ordinal weights, Moskowitz *et al*. (2000) selected $V2$ as the non-dominated alternative. Moskowitz *et al*. (2000) argued that this "expected" difference is because different approaches, different model assumptions and different criteria for filtering inferior alternatives were employed.

Even though there is some difference between these approaches, we do come up with many consistent solutions. First, our stochastic DEA either term $V2$ or $V4$ or V8 as most efficiency alternative, which is consistent with our simulation result in section 3 since simulation indicates $V2$, $V4$, $V6$ and $V8$, all have potential to be selected as non-dominated vendor. Second, our stochastic DEA models as well as the classical DEA in table 7 agree that $V1$, $V7$, $V9$ are frequently been filtered due to their poor performance represented in efficiency value. This verifies the strong diagnosing power in identifying the worst cases.

In order to compare our stochastic DEA result with classical DEA, we employ four classical DEA models to compute DEA efficiency without considering the variation in data, i.e. we only consider means in table 3. Table 7 presents the computed efficiencies using these four models: CCR without weight restriction, super CCR without weight restriction, CCR with weight restriction and super CCR with weight restriction. The super efficiency (SE) DEA model obtains individual reference functions for the efficient observations by removing the constraint related to the unit being evaluated from classical CCR model. In this way, DEA efficiency of the efficient DMUs is allowed to be greater than unity so that the discrimination area is expanded and the discrimination power is thus improved. Note, however, that the results for the standard model can always be recovered from the SE scores by setting all scores greater than one to unity. Andersen and Petersen (1993) provide more information about the SE DEA model.

Results from table 7 for the worst case are that vendor alternatives $\{V1, V6, V9\}$ are identified as inferior by CCR with weight restriction and super CCR with weight restriction. For super CCR without weight restriction a complete ranking is obtained as follows:

$$V5 \succ V6 \succ V3 \succ V4 \succ V8 \succ V7 \succ V2 \succ V1 \succ V9$$

The complete ranking provided by super CCR with weight restriction is different:

$$V2 \succ V3 \succ V8 \succ V4 \succ V5 \succ V7 \succ V6 \succ V1 \succ V9$$

Actual non-dominated alternatives are the set $\{V2, V4, V6, V8\}$. By non-dominated, we mean that there is a set of weights that would yield this option as preferred over all others. This implies that those that are dominated should never be selected by any set of weights (so super CCR without weight restriction fails, and CCR with weight restriction fails).

Note that although two of these four DEA models do provide a ranking order for all vendors, we do not recommend them for decision-making since they fail to consider the data variation or uncertainty embedded in the problem. To be specific, the restricted super CCR model never takes the standard deviation of the data into consideration in the computation process. This nature makes its inappropriate in our case. On the contrary, stochastic DEA provides a good tool to perform efficiency analysis by incorporating both inefficiency and stochastic error. Stochastic DEA also allows much flexibility for decision-makers to selected preferred vendors by use of the aspiration level and an allowable chance (risk) of failing to satisfy the constraint with which it is associated.

5. Conclusions

Vendor selection in supply chains by its nature involves the need to trade off multiple criteria, as well as the presence of uncertain data. When these conditions exist, stochastic dominance can be applied if the uncertain data are normally distributed. If not normally distributed, simulation modeling applies (and can also be applied if data is normally distributed).

DEA has been offered as an alternative approach. We have compared different stochastic versions of DEA. DEA can help to improve a performance measurement system in supply chain management. When the data is presented with uncertainty, stochastic DEA provides a good tool to perform efficiency analysis by handling both inefficiency and stochastic error. To employ the stochastic DEA vendor selection model, decision-makers are comfortable in choosing the aspiration level and an allowable chance (risk) of failing to satisfy the constraint with which it is associated. Models are insensible to different combinations of the aspiration level β and the allowable chance (risk) of failing to satisfy the constraint α.

Our preference is for simulation analysis, due to its flexibility in being able to cope with a variety of data distributions. Combined with SMART multi-criteria models, simulation can be the basis for complete ranking of alternatives, given decision-maker input of preference data.

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